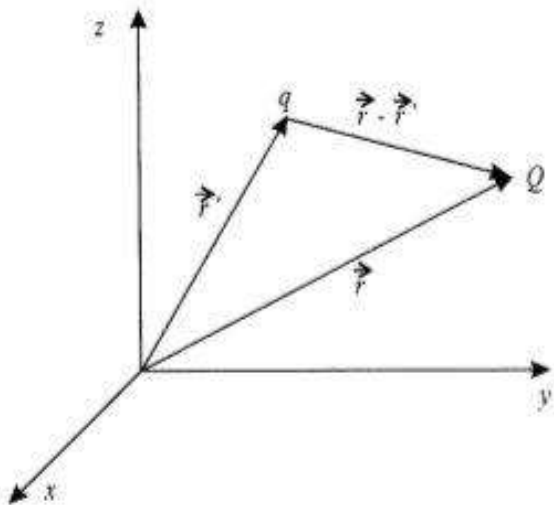


# REVISÃO

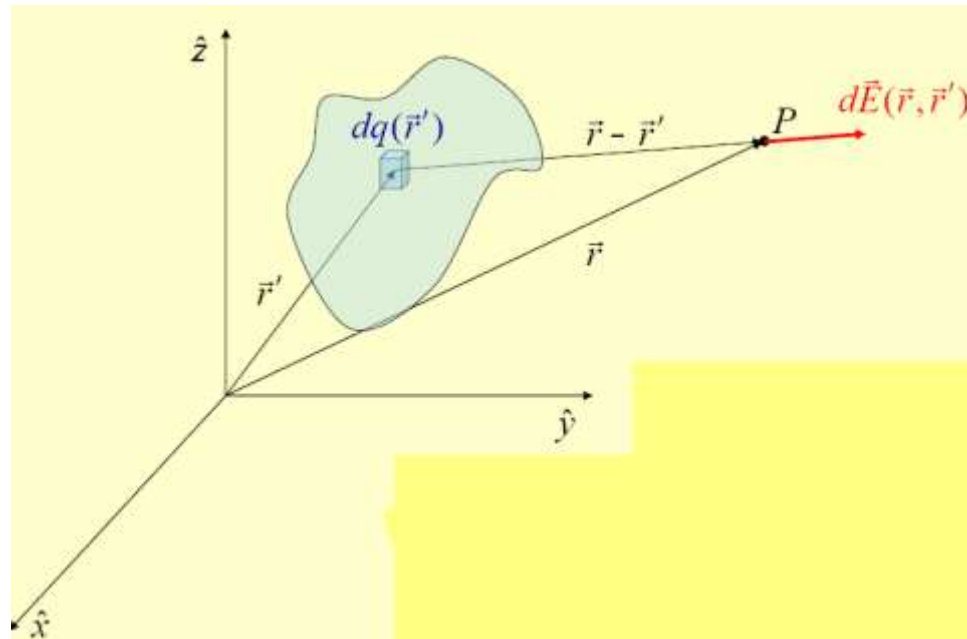
## Lei de Coulomb



$$\vec{F} = \frac{qQ}{4\pi\epsilon_0} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

# O campo elétrico

Distribuição contínua de cargas



$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{(\vec{r} - \vec{r}') dq}{|\vec{r} - \vec{r}'|^3}$$

Linha de carga:

$\lambda$  = carga/m

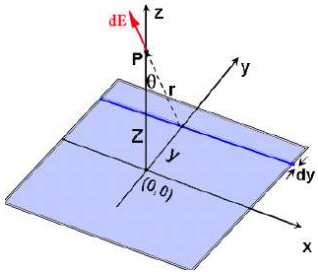


$$dq = \lambda dx$$

$$\vec{E} = \frac{2\lambda}{4\pi\epsilon_0} \frac{\vec{R}}{R^2}$$

$$|E| = \frac{2\lambda}{4\pi\epsilon_0} \frac{1}{R}$$

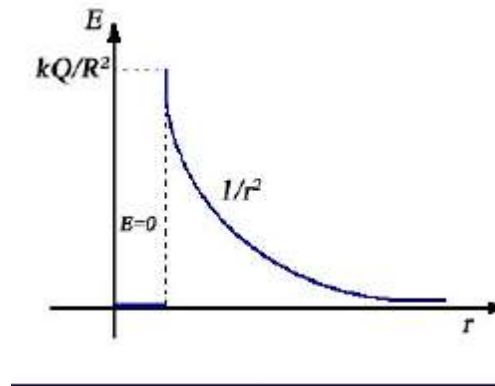
Plano infinito de carga  $dq = \sigma dS$



$$\vec{E} = \frac{\sigma}{2\epsilon_0} \frac{\vec{|z|}}{z} \quad |E| = \frac{\sigma}{2\epsilon_0}$$

Casca esferica de carga  $dq = \sigma dS$

Dentro  $E=0$  ; Fora  $\vec{E} = \frac{4\pi a^2 \sigma}{4\pi \epsilon_0} \frac{\vec{R}}{R^3} = \frac{Q}{4\pi \epsilon_0} \frac{\vec{R}}{R^3}$  ;  $|E| = \frac{Q}{4\pi \epsilon_0} \frac{1}{R^2}$



Esfera maciça de carga

**Volume de carga:**

$\rho = \text{carga/m}^3$



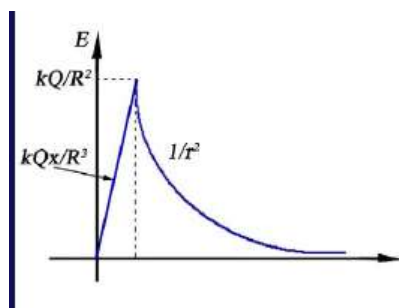
$$dq = \rho dV$$

Dentro

$$\vec{E} = \frac{\rho}{3\epsilon_0} \vec{R} = \frac{Q}{4\pi\epsilon_0 a^3} \vec{R};$$

Fora

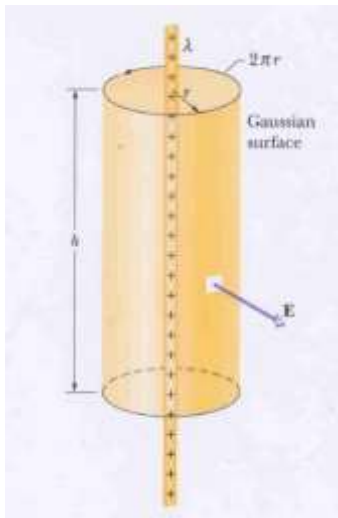
$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^3} \vec{R} ; |E| = \frac{Q}{4\pi\epsilon_0 R^2}$$



## Lei de Gauss

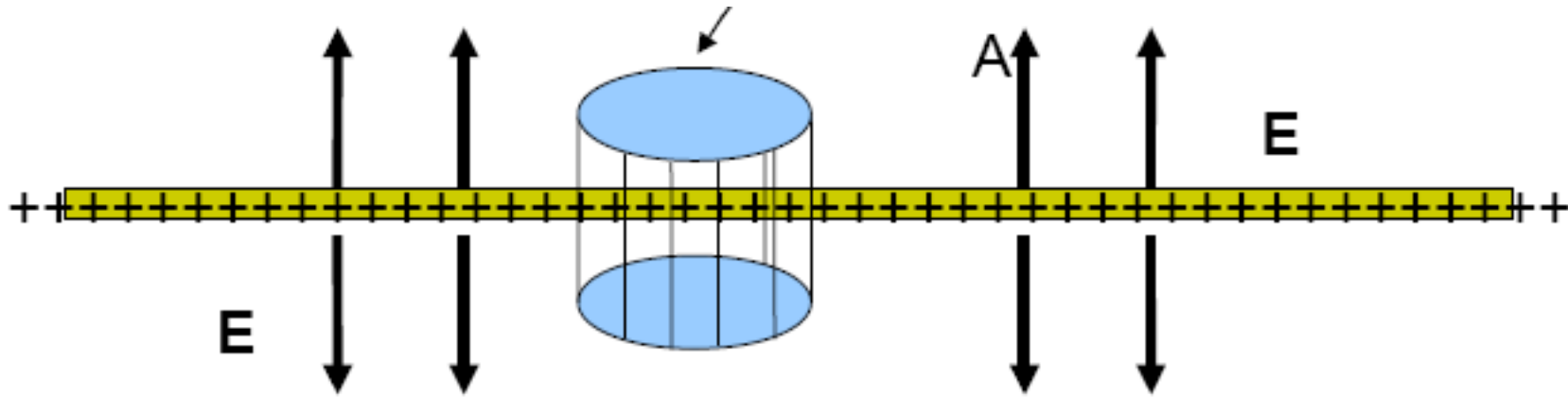
$$\int (\vec{E} * \vec{dS}) = \frac{Q_{total}}{\epsilon_0}$$

Linha de carga



$$\int (\vec{E} * \vec{dS}) = E(2\pi RL) = \frac{\lambda L}{\epsilon_0}; |E| = \frac{2\lambda}{4\pi\epsilon_0} \frac{1}{R}$$

Plano infinito de carga



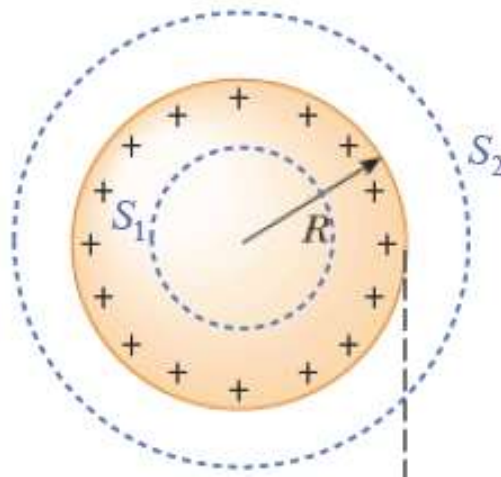
$$\int (\vec{E} * \vec{dS}) = E(S) + (-E)(-S) = 2ES = \frac{\sigma S}{\epsilon_0}; |E| = \frac{\sigma}{2\epsilon_0}$$

Casca esférica de carga

Dentro  $E=0$

Fora

$$\int (\vec{E} * \overrightarrow{dS}) = E(4\pi R^2) = \frac{Q}{\epsilon_0} ; |E| = \frac{1}{4\pi\epsilon_0} \frac{1}{R}$$



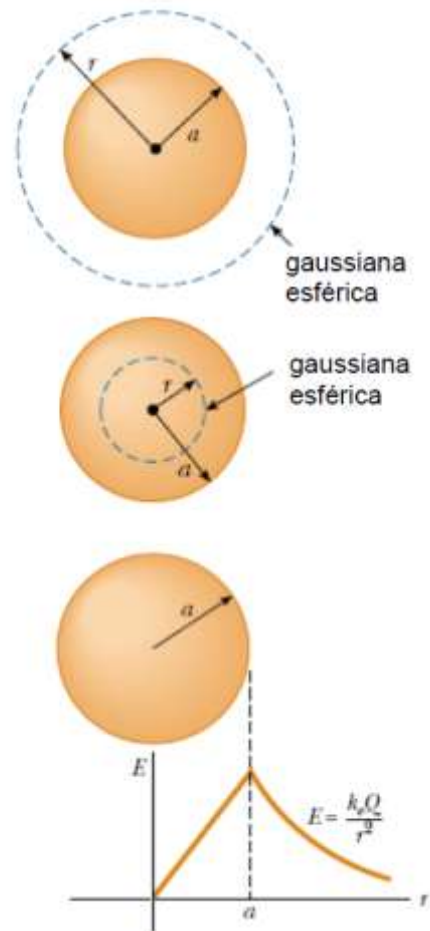


## Esfera maciça de carga

Dentro

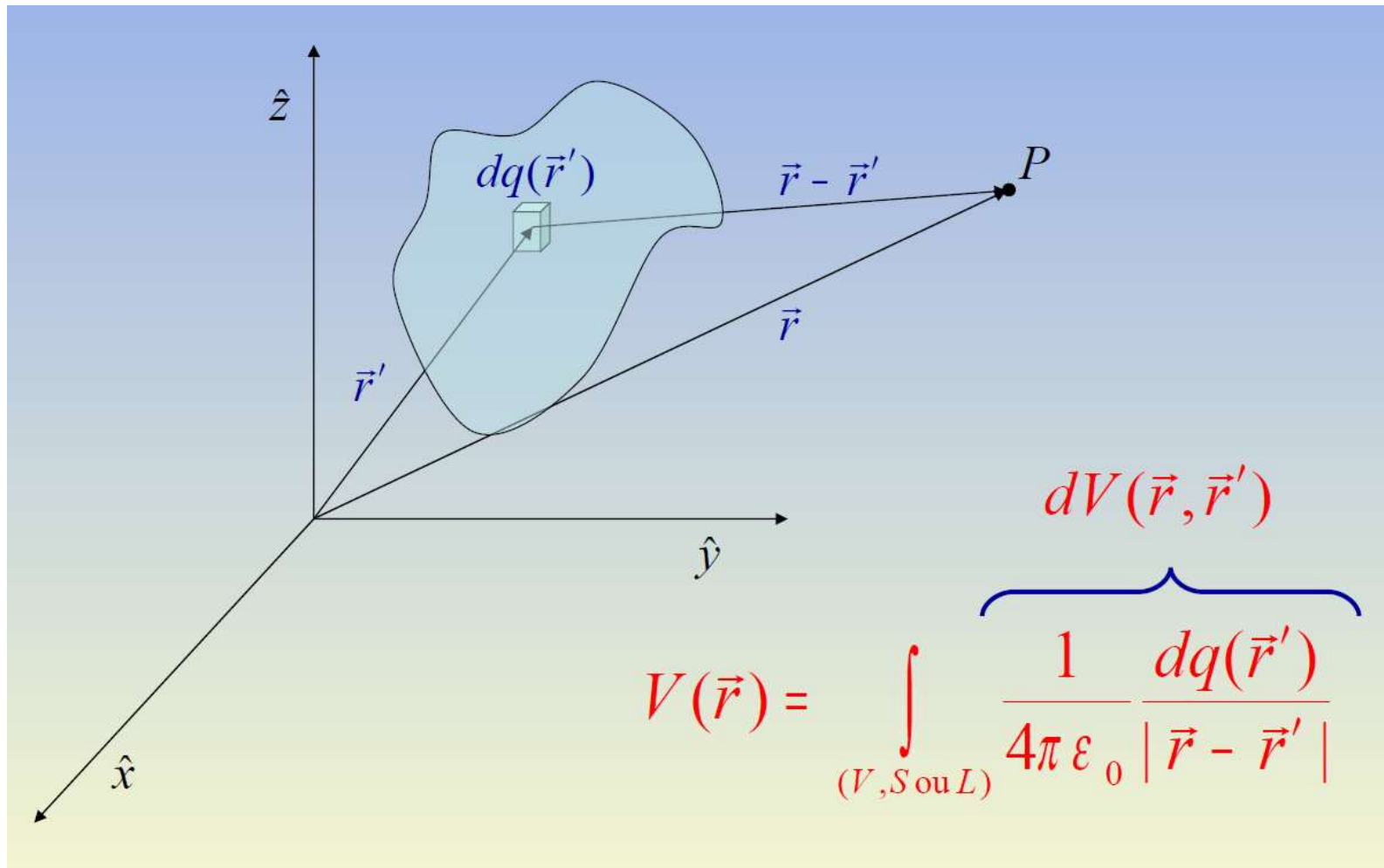
$$\begin{aligned}\int (\vec{E} * \vec{dS}) &= E(4\pi R^2) = \frac{Q}{\varepsilon_0} \\ &= \frac{1}{\varepsilon_0} \int_0^R \rho dV = \frac{1}{\varepsilon_0} \int_0^R \rho 4\pi R^2 dR = \frac{4\pi\rho R^3}{3\varepsilon_0}; |E| = \frac{\rho R}{3\varepsilon_0}\end{aligned}$$

# Fora



$$\begin{aligned}
\int (\vec{E} \cdot \vec{dS}) &= E(4\pi R^2) = \frac{Q}{\epsilon_0} \\
&= \frac{1}{\epsilon_0} \int_a^\infty \rho dV \\
&= \frac{1}{\epsilon_0} \int_a^\infty \rho 4\pi R^2 dR = \frac{4\pi \rho a^3}{3\epsilon_0}; \quad |E| = \frac{Q}{4\pi \epsilon_0 R^2}
\end{aligned}$$

# O Potencial Eléctrico



## Linha de carga

**Linha de carga:**  
 $\lambda$  = carga/m

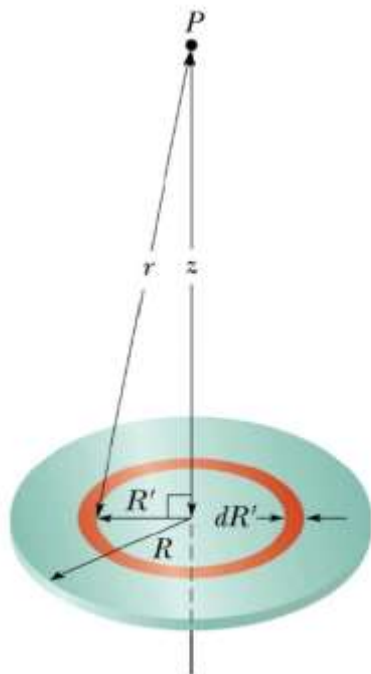


$dq = \lambda dx$

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln \frac{\sqrt{x^2 + a^2} + a}{\sqrt{x^2 + a^2} - a}$$

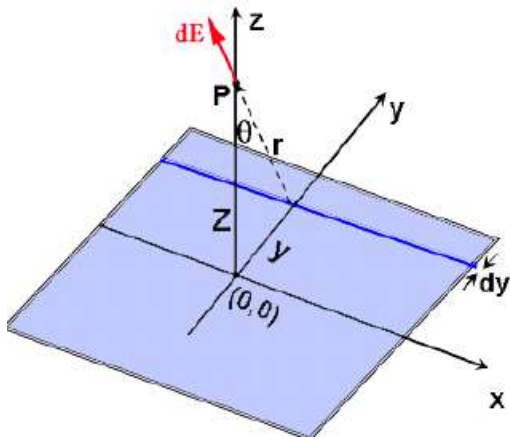
$$a \rightarrow \infty ; \Delta V = V(R_2) - V(R_1) = \frac{2\lambda}{4\pi\epsilon_0} \ln \frac{R_1}{R_2}$$

Disco de carga  $dq = \sigma dS$



$$V = \frac{\sigma}{2\epsilon_0} (\sqrt{z^2 + a^2} - |z|)$$

Plano infinito de carga  $dq = \sigma dS$



$$\Delta V = V(z) - V(0) = \frac{\sigma}{2\epsilon_0} \left( \sqrt{z^2 + a^2} - |z| \right) - \frac{\sigma}{2\epsilon_0} a =$$
$$- \frac{\sigma}{2\epsilon_0} |z|; \quad a \rightarrow \infty$$

## CÁLCULO DO POTENCIAL A PARTIR DO CAMPO ELÉTRICO

$$\begin{aligned} V(R) - V(\infty) &= - \int_{\infty}^R \vec{E} \cdot d\vec{l} ; \quad \vec{E} = -\vec{\nabla}V \\ &= -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k} \end{aligned}$$



Casca esférica de carga

Dentro

$$V(R) - V(\infty) = - \int_{\infty}^R \vec{E} \cdot \overrightarrow{dR} = \frac{Q}{4\pi\epsilon_0} \frac{1}{a}$$

Fora

$$V(R) - V(\infty) = - \int_{\infty}^R \vec{E} \cdot \overrightarrow{dR} = \frac{Q}{4\pi\epsilon_0} \frac{1}{R}$$

Esfera maciça de carga

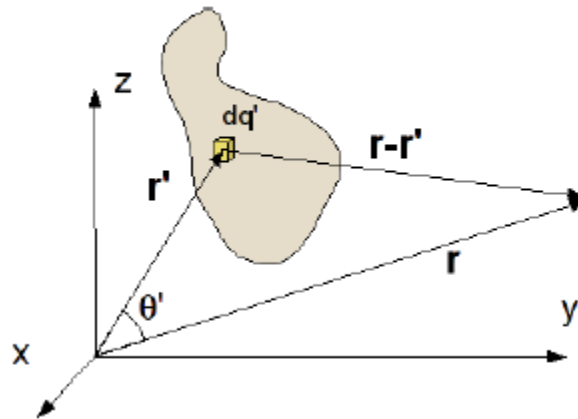
Dentro

$$V(R) - V(\infty) = - \int_{\infty}^R \vec{E} \cdot \overrightarrow{dR} = \frac{Q}{8\pi\epsilon_0} \left( 3 - \frac{R^2}{a^2} \right)$$

Fora

$$V(R) - V(\infty) = - \int_{\infty}^R \vec{E} \cdot \overrightarrow{dR} = \frac{Q}{4\pi\epsilon_0} \frac{1}{R}$$

Expansão Multipolar, Dipolo elétrico

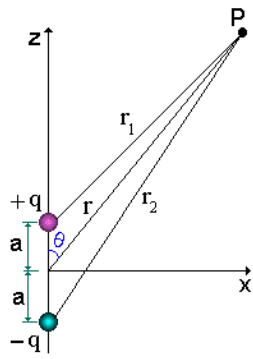


$$\begin{aligned}
V &= \frac{q}{4\pi\epsilon_0} \frac{1}{|\vec{R} - \vec{R}'|} = \frac{q}{4\pi\epsilon_0} \frac{1}{\sqrt{R^2 + R'^2 - 2RR'\cos\theta}} \\
&= \frac{q}{4\pi\epsilon_0} \frac{1}{R\sqrt{1 + \frac{R'^2}{R^2} - 2\frac{R'}{R}\cos\theta}} \\
&\approx \frac{q}{4\pi\epsilon_0} \frac{1}{R} \left( 1 + \frac{R'}{R} \cos\theta \right) = \frac{q}{4\pi\epsilon_0} \frac{1}{R} \left( 1 + \frac{(\vec{R}'\vec{R})}{R^2} \right) \\
&= \frac{q}{4\pi\epsilon_0} \frac{1}{R} + \frac{1}{4\pi\epsilon_0} \frac{(\vec{p}R)}{R^3}; \quad \vec{p} = q\vec{R}'
\end{aligned}$$

$$V_{total} = \sum V_i = \frac{1}{4\pi\epsilon_0} \frac{1}{R} \sum q_i + \frac{1}{4\pi\epsilon_0} \frac{(\vec{p}\vec{R})}{R^3}; \quad \vec{p} = \sum q_i \vec{R}_i' = \int \vec{R}' dq$$

$$Q = \sum q_i = 0; \quad \vec{E} = \frac{1}{4\pi\epsilon_0} \left( -\frac{\vec{p}}{R^3} + \frac{3(\vec{p}\vec{R})\vec{R}}{R^5} \right)$$

**Um dipolo elétrico é formado por duas cargas puntiformes**



$$V = \frac{1}{4\pi\epsilon_0} \frac{(\vec{p} \cdot \vec{R})}{R^3}$$

$$E_z = \frac{p_z}{4\pi\epsilon_0} \frac{1}{R^3} (3\cos^2\theta - 1); \quad E_x = 0; \quad E_y$$

$$= \frac{p_z}{4\pi\epsilon_0} \frac{3}{R^3} \cos\theta \sin\theta; \quad |E|$$

$$= \frac{p_z}{4\pi\epsilon_0} \frac{1}{R^3} \sqrt{3\cos^2\theta + 1}; \quad p_z = 2qa$$

## Energia Eletrostática

$$W = \frac{1}{2} \sum_{i=1}^n q_i \sum_{j=n \neq i}^n \frac{q_j}{4\pi\epsilon_0} \frac{1}{|\vec{R}_i - \vec{R}_j|}$$

$$W = \frac{1}{2} \int_0^{Volume} \varphi dq = \frac{\epsilon_0}{2} \int_0^{\infty} |E^2| dV; \quad \varphi - \text{potencial elétrico}; V - \text{volume}$$

Uma esfera uniformemente carregada

$$W = W_{dentro} + W_{fora} = \frac{1}{10} \frac{Q^2}{4\pi\epsilon_0} \frac{1}{a} + \frac{1}{2} \frac{Q^2}{4\pi\epsilon_0} \frac{1}{a} = \frac{3}{5} \frac{Q^2}{4\pi\epsilon_0} \frac{1}{a}$$

## Capacitância

$$C = \frac{Q}{V}$$

### Capacitor de placas paralelas

$$C = \frac{\varepsilon_0 S}{d}; \quad C = \frac{\varepsilon \varepsilon_0 S}{d}; \quad S - \text{area}, d - \text{distância entre as placas};$$

$\varepsilon - \text{constante dielétrica}$

### Capacitor cilíndrico

$$C = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{b}{a}\right)}; l \text{ é o comprimento de capacitor}$$

### Capacitor esférico

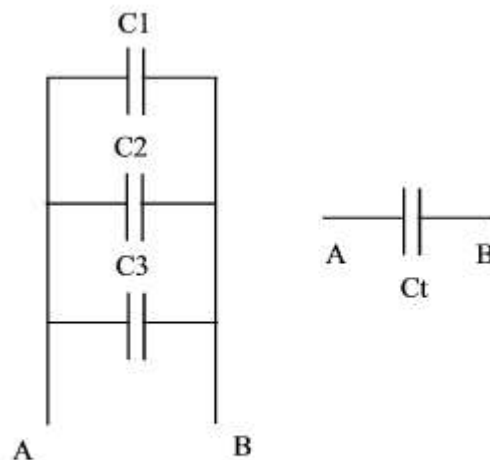
$$C = 4\pi\epsilon_0 \frac{ab}{(b - a)}$$

### Energia eletrostática armazenada

$$U = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

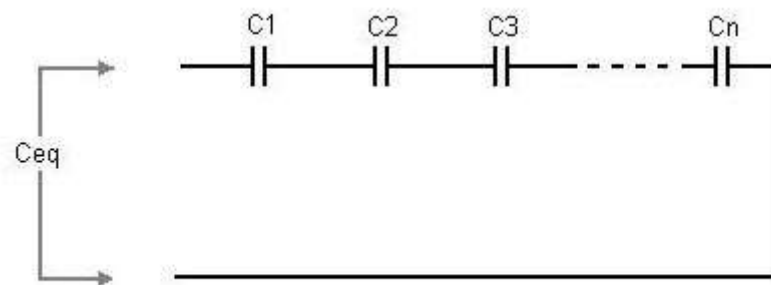


## Associação de capacitores paralelos



$$C = C_1 + C_2 + C_3 + C_4 + \dots C_n$$

## Associação de capacitores em série



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \dots + \frac{1}{C_n}$$

## Corrente elétrica, Lei de Ohm

$$I = \frac{dQ}{dt} = \frac{e^2 n \tau S}{m L} V$$

$$V = IR; R = \frac{m L}{e^2 n \tau S} = \rho \frac{L}{S}$$

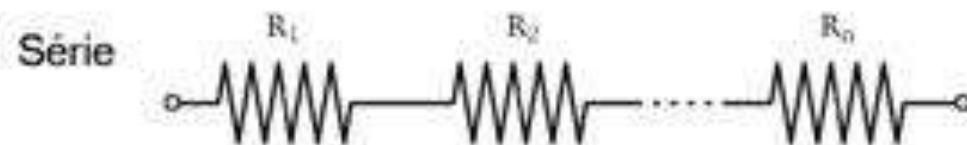
$$\rho = \frac{m}{e^2 n \tau} = \frac{1}{\sigma};$$

$R$  – *resistencia*,  $\rho$  – *resistividade*,  $\sigma$  – *condutividade*

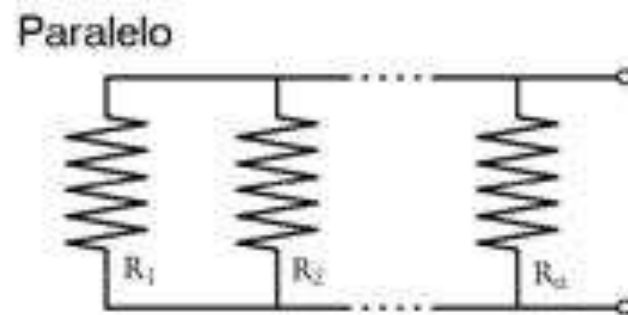
## O efeito Joule

$$P = \frac{dW}{dt} = IV = I^2 R = \frac{V^2}{R}$$

## Associação de resistores



$$R_t = R_1 + R_2 + R_3 + \dots + R_n$$



$$1/R_t = 1/R_1 + 1/R_2 + \dots + 1/R_n$$