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Seminar

# Fractional quantum Hall effect

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## Abstract

In this seminar I present the fractional quantum Hall effect (FQHE), which can be observed at low temperatures and high magnetic fields in two-dimensional electron systems (2DESs). After a brief overview of Hall effect, Landau levels and 2DESs, the FQHE and integer quantum Hall effect are presented and explained using Laughlin's gauge invariance and Jain's composite-particle approach, with more focus on FQHE specialties.

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## 1 Introduction

When an electrical conductor is exposed to a magnetic field perpendicular to the conductor, voltage transversal to an electric current direction is observed. The transversal voltage has linear dependence on the electric current and the magnetic field. As Edwin Hall was the one to discover this effect, it is called Hall effect.

Quantum Hall effect (QHE) is a term, describing the occurrence of transverse conductivity  $\sigma_{xy}$  quantization in two dimensional electron systems (2DESs) at fields on the order of 10 T and temperatures on the order of 1 K. The conductivity is quantized with values [1]:

$$\sigma_{xy} = \frac{1}{R_H} \nu, \text{ where } R_H = \frac{h}{e^2} \quad (1)$$

QHE covers two very different phenomena: integer quantum Hall effect (IQHE), where  $\nu$  from equation 1 is an integer and fractional quantum Hall effect (FQHE), where  $\nu$  is a fraction. They differ in interpretation as well - if IQHE is considered as a case of non-interacting electrons, FQHE results from the condensation of the two-dimensional electron gas. Even more stunning are the excitations of the ground state in FQHE - they were predicted to have fractional charge [2], which was later experimentally observed [3]. The Nobel Prize in Physics in 1998 was awarded to Störmer and Tsui for the discovery of the FQHE and to Laughlin for his theory [4].

The main focus of this seminar is to present FQHE, theory as well as some experimental features. I shall start with a few words on Hall effect. Further on I shall explain what makes all the difference in 2DESs compared to three dimensions. Finally composite-fermion approach will be presented as a theory explaining the observations and Laughlin wave function as a solution of many-body Hamiltonian for  $1/q$  states.

## 2 Hall effect

In a homogeneous and isotropic conductor electric current density  $\mathbf{j}$  and electric field  $\mathbf{E}$  are proportional:

$$\mathbf{j} = \sigma \mathbf{E},$$

where  $\sigma$  is the electrical conductivity. If the external magnetic field  $\mathbf{H}$  is applied, we rewrite the proportionality above as

$$j_i = \sigma_{ij} E_j,$$

where  $\sigma_{ij}$  is an ij-component of a rank two tensor, the conductivity tensor. The conductivity tensor, can be divided into symmetric and antisymmetric parts, where the former determines the Joule heat generation and the latter giving rise to the Hall effect [5]. Alternatively, resistivity tensor  $\rho$  can be defined to satisfy the equation

$$\mathbf{E} = \rho \mathbf{j},$$

thus being equal to the inverse of the conductivity tensor.

The Hall effect was discovered in 1879 by Edwin Hall. The novelty of the discovery was the voltage  $V_H$  across the current path, resulting from accumulation of charge on one side as compared to the other due to the current perpendicular to electric field, determined by the antisymmetric part of conductivity tensor. Hall deduced from his different experiments that voltage  $V_H$  is proportional to the magnetic field  $B$  and to the current  $I$  parallel to the electric field. [6]

Typical geometry for measurement of Hall effect is shown in Figure 1. Usually the longitudinal voltage  $V$  and transversal voltage  $V_H$  are determined, or their corresponding resistivity tensor components,  $\rho_{xx} = 1/\sigma_{xx} = E_x/j_x$  and  $\rho_{xy} = 1/\sigma_{xy} = E_y/j_x$  respectively. We chose the electric current vector  $\mathbf{j}$  to point along x axis and chose the direction, where  $V_H$  can be observed, as y axis.

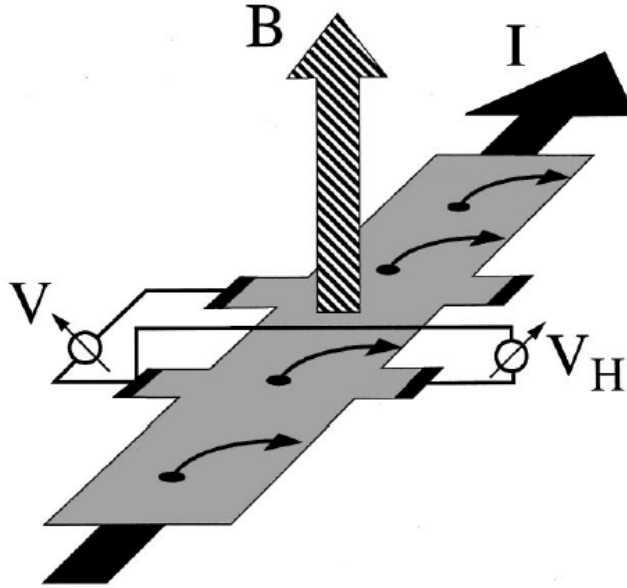


Figure 1: Geometry for measurement of the longitudinal voltage  $V$  and transversal Hall voltage  $V_H$ . Black curves represent the classical trajectories of electrons in the magnetic field [6].

It can be shown [1] that the Hall coefficient  $\mathcal{R}$ ,  $\mathcal{R} = E_y/Bj_x$ , can be written as

$$\mathcal{R} = -\frac{1}{nec}$$

for conductors with electron-like transfer, where  $n$  denotes the electron density. Thus, the Hall effect is frequently used to determine the carrier density of free electrons in electrical conductors.

In 1980 von Klitzing, Dorda, and Pepper measured the Hall effect in the inversion layer at Si/SiO<sub>2</sub> interface at low temperatures and high fields. Instead of reproducing the linear dependence on the magnetic field, they observed that the transverse conductivity  $\sigma_{xy}$  was quantized [1]. Which part of their experiment was the one to produce such drastic changes compared to Edwin Hall results?

### 3 Two-dimensional electron systems

While Halls sample was a sheet of gold leaf [6], which is a three-dimensional electron system, von Klitzing, Dorda, and Pepper used a sample, where a two-dimensional electron gas is present. As electrons are restricted to move only in two dimensions, their behaviour is bound to be different than in three-dimensional systems, for instance, as Landau pointed out, that motion in a plane perpendicular to a uniform magnetic field has discrete energy levels, called Landau levels [7].

#### 3.1 Landau quantization

Consider a particle of mass  $m$ , spin  $\mathbf{s}$  and magneton  $\mu$  confined between two planes, for instance an electron in a 2DES. For the vector potential of the uniform magnetic field, taken in the form

$$A_x = -By, \quad A_y = A_z = 0,$$

the Hamiltonian is [7]:

$$\hat{H} = \frac{1}{2m} \left( \hat{p}_x - \frac{eBy}{c} \right)^2 + \frac{\hat{p}_y^2}{2m} - \mu \hat{s}_z B. \quad (2)$$

We seek the solution of the stationary Schrödinger equation in the form

$$\psi = e^{ip_x x/\hbar} f(y).$$

Using this ansatz in Schrodinger equation for Hamiltonian in equation 2, we obtain:

$$f''(y) + \frac{2m}{\hbar^2} \left[ (E + \mu s_z B) - \frac{1}{2} m \omega_c^2 (y - y_0)^2 \right] f(y) = 0, \quad (3)$$

where we defined cyclotron frequency  $\omega_c$  as

$$\omega_c = \frac{eB}{mc}$$

and  $y_0$  as

$$y_0 = -\frac{cp_x}{eB}.$$

Formally equation 3 represents a Schrödinger equation for a harmonic oscillator centred about  $y_0$  and oscillating with frequency  $\omega_c$ . Following the harmonic oscillator analogy we write the energy levels of a particle in a uniform magnetic field as

$$E_{\nu_L, k_x, k_z} = (\nu_L + 1/2) \hbar \omega_c + \mu s_z B,$$

where the integer  $\nu_L$  labels the Landau levels. Even in three dimensions the motion in the plane perpendicular to the field is still quantized. However, due to the velocity in the direction of the field taking any value, motion in an arbitrary direction is not quantized. If the particle is restricted to move only in one dimension, Landau quantization does not arise.

Considering that our sample is finite, we obtain the degeneracy of energy states in sample with total area of  $A$  as [1]

$$N = \frac{BA}{\phi_0} = \frac{\Phi}{\Phi_0},$$

where  $\Phi_0 = \frac{hc}{e}$  is the magnetic flux quantum. We can further define the Landau-level filling factor  $\nu$  as the ratio of the number of free electrons to the degeneracy of energy states. For 2D electron density of  $n$ , we obtain

$$\nu = \frac{An}{N} = \frac{n\hbar c}{eB}.$$

In a realistic two-dimensional electron system (2DES) impurities are always present and as such, completely discrete density of states is not observable even in the purest of samples. Instead, there are disorder-induced localized states in between the Landau level peaks as shown in Figure 2.

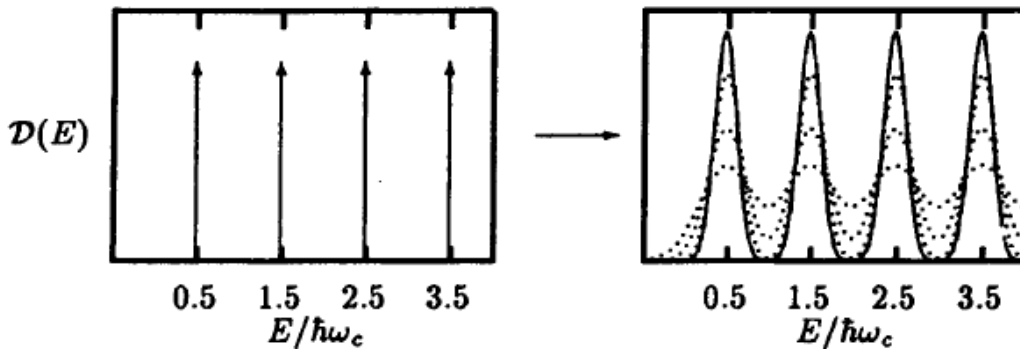


Figure 2: Density of energy states with a large number of states at energies  $\hbar\omega_c(\nu + 1/2)$  corresponding to Landau quantization in 2DESs. The presence of impurities on the right side broadens the peaks and produces localized states between the peaks [10].

### 3.2 Creation of a two-dimensional electron system

Constructing a 2DES can be done in a three-dimensional world with confining electrons to the interface between two substances, thus restricting electron movement in direction perpendicular to the interface. The QHE was discovered in a 2DES where electrons were confined to the interface between two different semiconductors. While Klitzing, et al. used the Si MOSFET to create a 2DES, Störmer, Tsui, and Gossard did this at a GaAs/AlGaAs heterojunction obtaining much higher mobilities of the electrons. Both types of 2DES are schematically shown in Figure 3. They used samples consisting of undoped GaAs, undoped  $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ , Si-doped  $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ , and Si-doped GaAs single crystals, each around 500 Å thick, sequentially grown on GaAs substrates using molecular beam epitaxy techniques [8], which are high-vacuum evaporation techniques allowing building up solids one layer at a time.

GaAs and AlGaAs interfaces are frequently used as there is nearly no lattice distortion incurred by placing layers of GaAs and AlGaAs together. This is due to GaAs and AlAs having very similar crystal structure, with lattice constants of 5.63 Å and 5.62 Å respectively and both adopting the zincblende structure, therefore by replacing some gallium atoms with aluminium ones lattices change negligibly. [1]

Gap energies for  $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$  and GaAs differ and give rise to band bending at the junction between  $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$  and GaAs layers as shown in Figure 4(A). When doping of both sides of

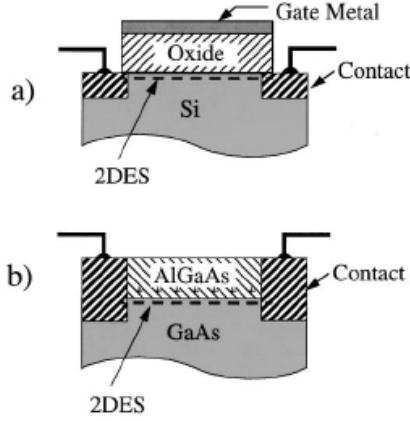


Figure 3: Schematic drawings of a two dimensional electron gas in (a) a silicon MOSFET transistor and (b) a GaAs/AlGaAs heterojunction [6].

the junction is heavy enough, chemical potential  $\mu$  can rise above the notch in the potential, thus creating a small region, occupied even at zero temperature, called inversion layer, as shown in Figure 4(C). The notch in the potential can be also interpreted to present a one-dimensional attractive triangular potential for electrons, always having at least one bound eigenstate. The potential barriers in the vicinity of heterojunction are on the order of 0.1 eV, thus restricting the temperatures to a few kelvin or less, if only the ground state was to have significant occupation. By creating a heterojunction between  $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$  and GaAs and keeping the temperature low enough, the electrons are trapped in the one-dimensional triangular potential, thus being confined to the interface between semiconductors and giving rise to a two-dimensional electron gas.

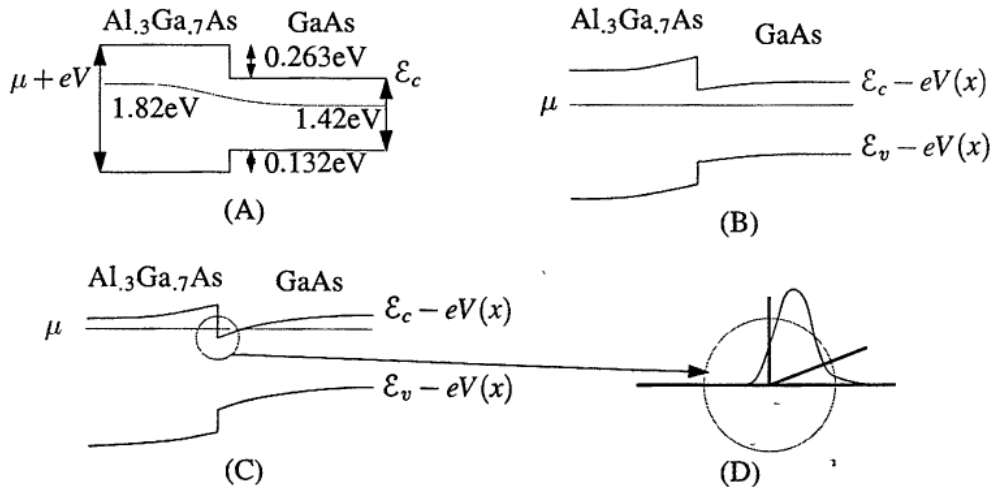


Figure 4: (A) and (B) schematic picture of junction between  $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$  and GaAs semiconductors. (C) In case of heavy doping chemical potential  $\mu$  can rise above conduction band edge, which is enlarged in (D), where a bound-state wave function is sketched. The junction plane is projected to one dimension for better clarity. [1]

Electron scattering is undesirable, as it obscures the mutual electron interactions and in-

teractions with the magnetic field. Scattering occurs on impurities, such as lattice distortions, impurities introduced with doping, and on phonons. The latter part of scattering can readily be minimized by cooling the sample, while the former part can be taken care of with appropriate choice of materials. As the heterojunction mentioned above has very small lattice distortions, only scattering on doped impurities needs to be prevented. This is done by introducing the doped Si atoms around  $0.1 \mu\text{m}$  away from the interface between  $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$  and GaAs semiconductors. This distance assures little scattering as well as transfer of electrons to inversion layer. [6]

The quality of samples containing 2DESs is usually described with electron mobility. Bulk GaAs has a mobility on the order of  $10^3 \text{cm}^2/\text{Vsec}$ , while recent doped samples achieved mobilities on the order of  $10^7 \text{cm}^2/\text{Vsec}$  at temperatures around 1 K. For illustration, mobility of  $2 \times 10^7 \text{cm}^2/\text{Vsec}$  corresponds to approximately 0.2 mm mean free path [6].

## 4 Fractional quantum Hall effect

In their article [8] Tsui, Stormer, and Gossard reported "some striking, new results on the transport of high-mobility, 2D electrons, in GaAs-AlGaAs heterojunctions in the extreme quantum limit, when the lowest-energy, spin-polarized Landau level is partially filled". The striking result was a dip in the diagonal part of the resistivity tensor  $\rho_{xx}$  and a plateau in Hall resistivity  $\rho_{xy}$  at the  $1/3$  filling factor of the first Landau level. Integer quantum Hall effect, discovered some time before Tsui, Stormer and Gossard conducted their experiment, with the same characteristics, a valley in  $\rho_{xx}$  and a plateau in  $\rho_{xy}$ , was only observed at full fillings of Landau levels.

As the filling factor  $\nu$  is determined with the number of free electrons in the sample as well as with the degeneracy of energy states, one can readily change the degeneracy of Landau levels by varying the strength of the magnetic field  $B$  and thus change the filling factor. Higher Landau levels have higher energy, therefore by raising the magnetic field higher levels start to lose electrons due to the change in degeneracy. Having localized states present as a consequence of impurities creates a range of  $B$  for which all the extended states in the lower-lying Landau level remain filled, while the higher-lying is completely empty (in a perfect sample with no impurities, the lower-lying level would start to lose the electrons as soon the higher-lying one was completely empty).

The occupation of the Landau levels not changing over a range of  $B$  corresponds with the formation of the energy gap, which in turn corresponds to the plateaus in Hall resistance. As the ranges of magnetic field strengths, where the lower Landau level stays filled, while the higher one is empty, lie at integer filling factors, this phenomenon is called integer quantum Hall effect. The plateau, which was observed forming at  $\nu = 1/3$  as shown in Figure 5, therefore could not have been the result of IQHE. The newly found phenomenon was later named fractional quantum Hall effect (FQHE) for occurring at fractional fillings of Landau levels and its underlying physics is substantially different from the physics underlying the IQHE.

The basic explanation for plateaus in Hall voltage in FQHE is the formation of the quantum liquid as a result of electrons trying to minimize the Coulomb repulsion. More detailed explanations will be provided further on in the seminar.

### 4.1 Laughlin's gauge argument

Quantum Hall effect describes Hall voltage and Hall resistance being quantized. Using gauge invariance argument Laughlin has derived the correct values for plateaus in Hall resistivity  $\rho_{xy}$  for IQHE by his gedanken experiment [9].

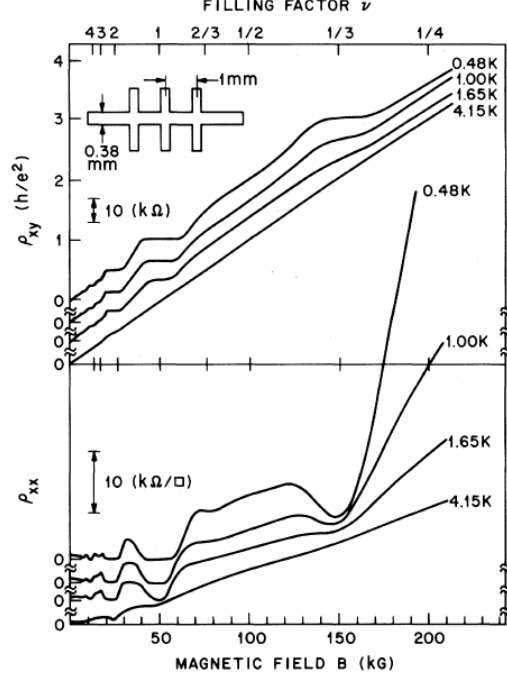


Figure 5: Measurements made by Tsui, Stormer, and Gossard, showing formation of a plateau in Hall resistivity  $\rho_{xy}$  at  $1/3$  filling of the first Landau level, with diagonal part of the resistivity tensor  $\rho_{xx}$  having a dip at the same filling factor. Sample geometry is shown in the top left corner. [8]

Let us follow Laughlin's train of thought to derive the quantum of resistance [9, 10]: As shown in Figure 8, take a two-dimensional metal strip bent into a loop pierced everywhere perpendicular to the surface by the magnetic field  $B$ .

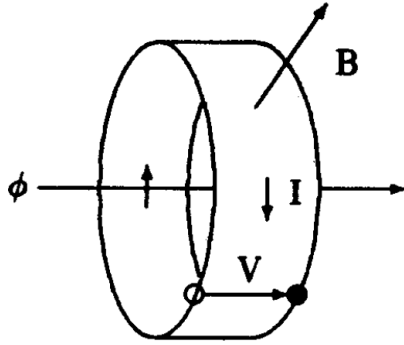


Figure 6: Diagram of metallic loop used in gauge invariance argument for QHE. [9]

For each electron adopt the Landau gauge as well as Hamiltonian from Equation 2, while adding a term covering the transversal electric field  $eE_0y$  to the Hamiltonian. The resulting energy eigenvalues are

$$\mathcal{E}_{\nu_L, k} = (\nu_L + \frac{1}{2})\omega_c + eE_0y_0 + \frac{1}{2}m \left( \frac{cE_0}{B} \right)^2,$$

where  $y_0$  represents the centre of a harmonic oscillator, as in Landau quantization.  $y_0$  being the



only term to change with the increase of the vector potential in the manner

$$y_0 \rightarrow y_0 - \frac{\Delta A}{B},$$

the energy  $\mathcal{E}_{\nu L, k}$  increases linearly with  $\Delta A$ .

The carrier states can be divided into two classes. One are the localized states, which are not continuous around the loop, and are unaffected by the application of the magnetic flux  $\Phi$  to the first order. The second one are the extended states, which are continuous around the loop, and as they enclose the magnetic flux, their energy can be changed. Addition of the magnetic flux quantum therefore maps the system into itself [11] and changes  $y_0$ , in the absence of the disorder, by

$$-\frac{\Delta A}{B} = -\frac{\Phi_0}{BL} = -\frac{\Phi_0 d}{BLd} = -\frac{d}{N},$$

where we denoted the width of the strip as  $d$  and  $N$  represents the degeneracy of Landau levels. The latter can be interpreted as evolving of a wave function into its neighbour with the overall effect of one electron per Landau level being transported to the other edge of the loop. Though in the system with small amount of disorder the mechanism of transfer is different, the number of transferred electrons stays the same [10]. The total current  $I$  around the loop can be obtained as a derivative of the total energy  $U$  with respect to vector potential or, for large enough loop as a differential

$$I = c \frac{\Delta U}{\Delta \Phi}$$

with the denominator  $\Delta \Phi$  being the magnetic flux quantum. All the energy increase is due to the transfer of  $\nu$  electrons from one edge of the loop to the other, overcoming the voltage  $V$ . The current is therefore

$$I = c \frac{\nu e V}{\Phi_0} = \frac{\nu e^2}{h} V,$$

resulting in Hall resistivity of

$$\rho_{xy} = \frac{1}{\nu} \frac{h}{e^2} = \frac{R_H}{\nu}, \nu \in \mathbb{N}. \quad (4)$$

The theoretical value differs from the measured values in the plateaus to a few parts per billion, thus leading to the quantum of resistance  $R_H = 25,813 \text{ k}\Omega$  being set as a new resistance standard in 1990 [6]. As the gauge invariance argument can be used for a general explanation for FQHE as well [1], the plateaus in resistivity will be at

$$\rho_{xy} = \frac{1}{\nu} \frac{h}{e^2} = \frac{R_H}{\nu}, \nu = \frac{p}{q} \in \mathbb{Q}^+. \quad (5)$$

## 4.2 Laughlin's trial wave function

While the phenomenology behind both of the QHE is practically the same, the cause for the energy gap needed for the phenomena to occur is very different. While in IQHE this gap arises from the formation of Landau levels and localized states due to impurities at Fermi level, the main cause for FQHE is the Coulomb interaction between electrons. Thus rigidity of the electron system in FQHE is imparted through the preferred density of electrons, which are discrete, to achieve minimal repulsion.

Laughlin's main contribution to the FQHE was his trial wave function, shown schematically on Figure 7, proposed in his article [2]. His prototype ground state

$$\Psi_m(z_1, \dots, z_N) = \prod_{j < k}^N (z_j - z_k)^m \exp \left( -\frac{1}{4l^2} \sum_j^N \|z_j\|^2 \right),$$

where  $m$  is an odd number, in Laughlin's case 3,  $l = \sqrt{\frac{\hbar c}{eB}}$  is the magnetic length<sup>1</sup> and  $z_j = x_j + iy_j$  is denoting the location of the  $j$ th particle as a complex number, is the approximate solution of the model Hamiltonian including Coulomb interaction between electrons  $v(\vec{r}_j - \vec{r}_k)$  and the ion potential  $V_{ion}(\vec{r}_j)$

$$\mathcal{H} = \sum_j^N \left\{ \frac{1}{2m} \left( -i\hbar\vec{\nabla}_j - \frac{e}{c}\vec{A}(\vec{r}_j) \right) + V_{ion}(\vec{r}_j) \right\} + \sum_{j<k}^N v(\vec{r}_j - \vec{r}_k).$$

The ion potential is present only to achieve stability of the system with neutralizing the electron interaction. Calculating the probability density as

$$\|\Psi_m(z_1, \dots, z_N)\|^2 = e^{-\beta\Phi(z_1, \dots, z_N)},$$

and choosing  $\beta = 1/m$ , one can obtain the potential energy  $\Phi(z_1, \dots, z_N)$

$$\Phi(z_1, \dots, z_N) = -2m^2 \sum_{j<k}^N \ln|z_j - z_k| + \frac{m}{2l^2} \sum_j^N |z_j|^2.$$

The first term can be described as coulomb repulsion of particles of "charges"  $m$  in two dimensions, while the second one represents a uniform "charge" density  $1/2\pi l^2$  being attracted to the origin. The particles need to have density  $\rho = 1/2m\pi l^2$ , if local electrical neutrality is to be present.

It can be readily seen, why the proposed wave function could work well. Having an odd  $m$  the wave function is anti-symmetrical to the particle exchange, which is required due to electrons being fermions. Furthermore, such a state exhibits the preference of having the particle density of  $\rho = 1/2m\pi l^2$ , which leads to the formation of the plateaus in transversal resistivity  $\rho_{xy}$ .

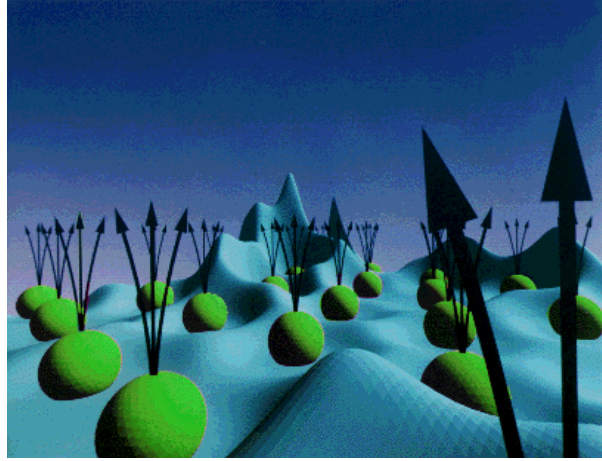


Figure 7: Charge distribution of a prototype electron for the  $\nu = 1/3$  Laughlin wave function. The green spheres represent electrons, while the black arrows represent flux quanta [12].

### 4.3 Partially charged excitations

Laughlin also proposed that the elementary excitations of the FQHE state carry fractional charge [2]. The excitations were generated in a though experiment by piercing the 2DES with

<sup>1</sup>the radius at which the charge would circle in presence of the magnetic field B

an infinitely thin solenoid and adiabatically passing a flux quantum  $\Phi_0$  through the solenoid. Similar to the thought experiment on gauge invariance passing of a flux quantum  $\Phi_0$  results in mapping of a system into itself. As the geometry is different Landau orbits far away from the solenoid are translated toward it [10]. The net effect is taking the average charge per state at infinity and depositing it near the solenoid as represented in Figure 8. Removing the solenoid will give us an exact excited state of the original Hamiltonian, carrying the above mentioned charge.

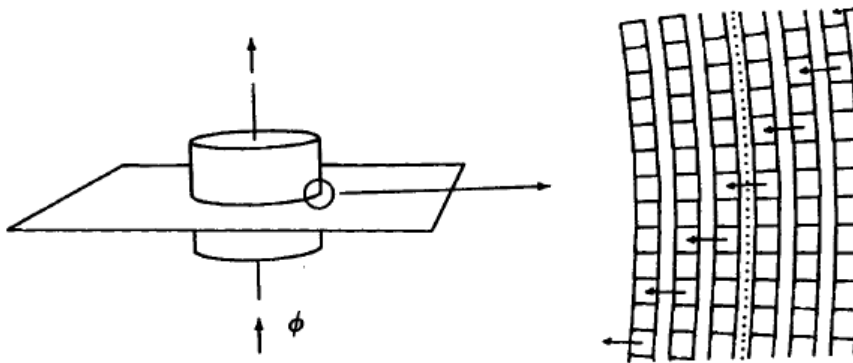


Figure 8: Illustration of the thought experiment demonstrating the partially charged excitations. The arrows on the right picture show the translation of the Landau levels [10].

Taking into account the preferred densities of  $1/2\pi ml^2$ , where  $m$  is an odd integer, corresponding in an average of  $e/m$  charge per state at infinity, the elementary excitations carry a charge of  $e/m$ .

#### 4.4 Composite-fermion approach

Trial wave function proposed by Laughlin [2] only covers the filling factors of type  $1/p$ . Other theories emerged later, such as hierarchical scheme by Haldane [13] and Halperin [14] and composite-fermion approach proposed by Jain [15], which try to provide a microscopic explanation for other observed states, not only for  $1/p$  states. I shall present the latter theory as it proposes a unified scheme for both FQHE and IQHE.

We have seen that the filling factor  $\nu$  plays a very important role in QHE. It determines the behaviour of the 2DES, namely whether it manifests IQHE or FQHE. Jain [15] proposes that the inverse of the filling factor, namely ratio of the total number of flux quanta to the total number of electrons, is the important parameter.

Laughlin [16] and Arovas et al. [17] observed the equivalence, in a mean field sense, of a uniform liquid of electrons in a magnetic field with the average flux per electron area of  $m\Phi_0$  and of a uniform liquid of electrons, each carrying with it a flux of  $m\Phi_0$ . As an analogy to type II superconductors, the magnetic field  $B$  piercing the electron gas can be viewed as forming of vortices or flux tubes within the liquid of electrons, one per flux quantum  $\Phi_0$ , as shown as a part of Figure 9. The electronic charge within the vortex is zero at its centre and average surrounding charge density at its edge. As such vortex is basically the same as the absence of electrons it is particularly beneficial to put a vortex on top of an electron due to Coulomb repulsion. If enough vortices are present, placing more of them onto the electron further reduces the repulsive energy. Putting a vortex onto an electron is the same as the attachment of a flux quantum, generating the vortex, to the charge carrier thus creating new entities called composite particles (CP).

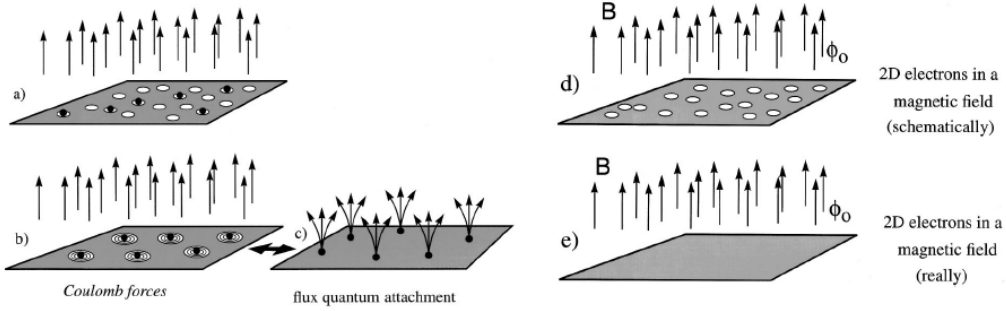


Figure 9: Schematic drawing of electron vortex attraction at Landau-level filling  $\nu = 1/3$ . a) The Pauli principle is satisfied by placing one vortex onto each electron. b) Placing more vortices onto each electron further reduces Coulomb repulsion. c) Vortex attachment can be viewed as the attachment of magnetic flux quanta to the electrons, creating composite particles. d) and e) show the 2DES in a magnetic field described with vortices [6].

Electrons are fermions and generate a  $-1$  phase shift with the exchange of the position of two electrons. Exchange of each flux quantum  $\Phi_0 = \frac{hc}{e}$  multiplies the wave function by an additional  $-1$  [6]. The statistics of CP therefore depend on, as Jain proposed, the number of flux quanta per charge carrier. If the magnetic field generates an even number of flux quanta per charge, the resulting CP obey the Fermi-Dirac statistics, hence can be called composite fermions (CFs), while an odd number of flux quanta per charge gives rise to composite bosons (CB).

The main idea of this approach is that, since the relevant correlations between CFs are the correlations due to their Fermi statistics, the FQHE for electrons is a manifestation of the IQHE for CFs [15].

The leftover vortices, which haven't been used for formation of CF, represent an effective field  $B^*$  of

$$B^* = B - 2mn\Phi_0,$$

where  $n$  denotes two-dimensional carrier density. Composite fermions form Landau-like levels in  $B^*$  with the filling factor  $\nu^* = \frac{n\hbar c}{eB^*}$ . In analogy with electrons for integer  $\nu^*$  IQHE for CF manifests, thus producing plateaus in Hall resistivity when filling factor for electrons  $\nu$  satisfies the demand

$$\nu = \nu^* \frac{B^*}{B} = \frac{\nu^*}{2m\nu^* \pm 1}, \quad (6)$$

where  $\pm$  once again denotes the direction of  $B^*$  compared to  $B$ .

As many of the states from Equation 6 have been observed thus far as shown on Figure 10, the theory has been back up by experiments quite well. Still this theory does not account for a peculiar state with  $\nu = 5/2$ , which was observed to exhibit all the FQHE characteristics [18] as shown on Figure 11, despite having an even denominator.

It is speculated that for FQHE features of the  $\nu = 5/2$  state pairing of CF produces a novel many-particle ground state. This pairing is thought to be analogous to formation of Cooper pairs and their subsequent condensation into a superconducting state. Unfortunately no experiments were made to confirm this speculation, so it is still unclear whether pairing of CFs is the right way to explain this phenomenon. [6]

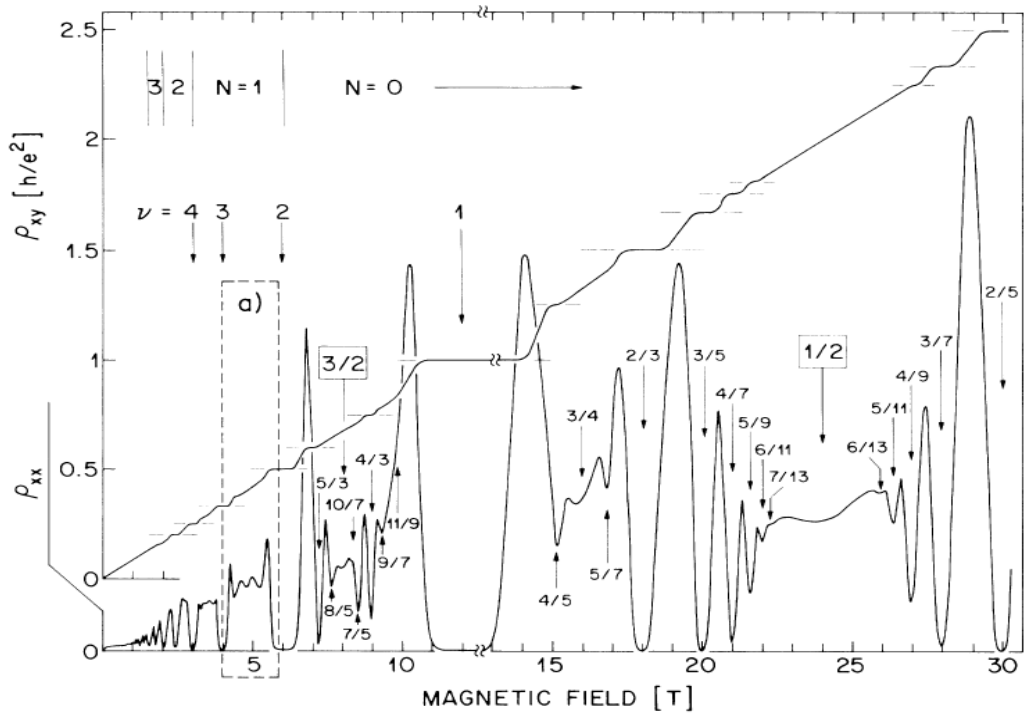


Figure 10: Overview of diagonal resistivity  $\sigma_{xx}$  and Hall resistance  $\sigma_{xy}$  of a GaAs/AlGaAs heterostructure at temperatures  $\approx 150$  mK. Filling factor  $\nu$  and Landau levels  $N$  are indicated. [18]

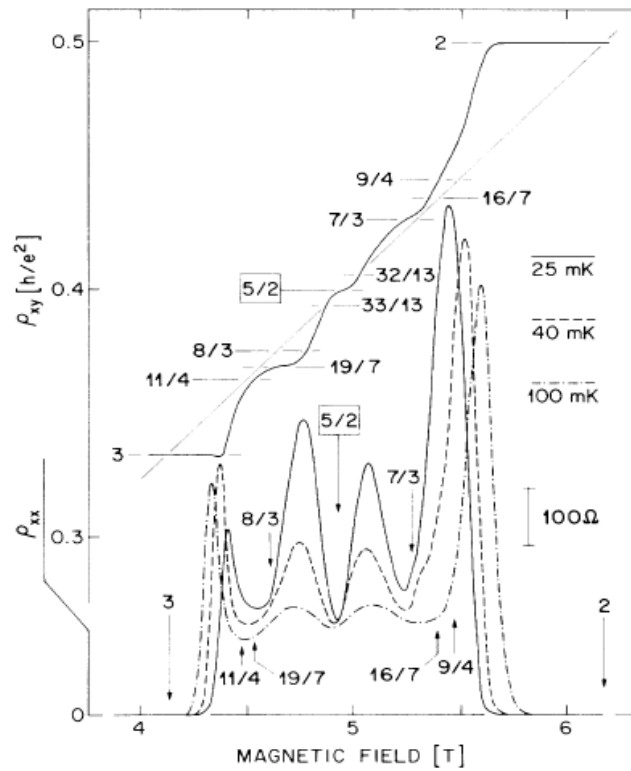


Figure 11: Diagonal resistivity  $\sigma_{xx}$  and Hall resistance  $\sigma_{xy}$  (enlarged section a) of Figure 10). Filling factors  $\nu$  are indicated in  $\sigma_{xx}$ , while quantum numbers  $p/q$  are shown in  $\sigma_{xy}$ . [18]

## 5 Conclusions

While paving our way to fractional quantum Hall effect, some important physical phenomena and discoveries were mentioned. We reviewed our knowledge of electrical conduction and Hall effect, rediscovered Landau quantization, where energy is quantized in a two-dimensional electrons system exposed to magnetic field, and we got to know how the high-mobility two-dimensional electrons systems are made in our three-dimensional world. It was also explained why and when do the electrons in two-dimensional electrons systems of high mobility show quantized Hall resistivity and minima in diagonal resistivity. In other words, the theoretical framework for quantum Hall effect was presented, namely the Laughlin's gauge invariance argument as a way to explain the value of quantum of resistance and composite-fermion approach as a microscopic theory of the FQHE. In the seminar the somewhat counter-intuitive excitations with fractional charge and the  $\nu = 5/2$  state were presented as a manifestation of the FQHE.

The discovery of IQHE brought a new standard for electrical resistivity and a method of precise measurement of structure constant. FQHE remains influential in theories about topological order, while some parts of the theoretical framework for FQHE are of a particular theoretical importance, such as composite particles [19].

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