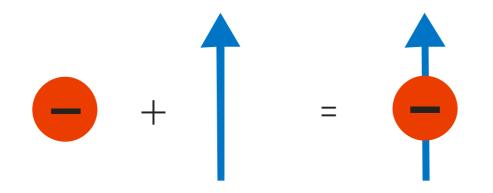
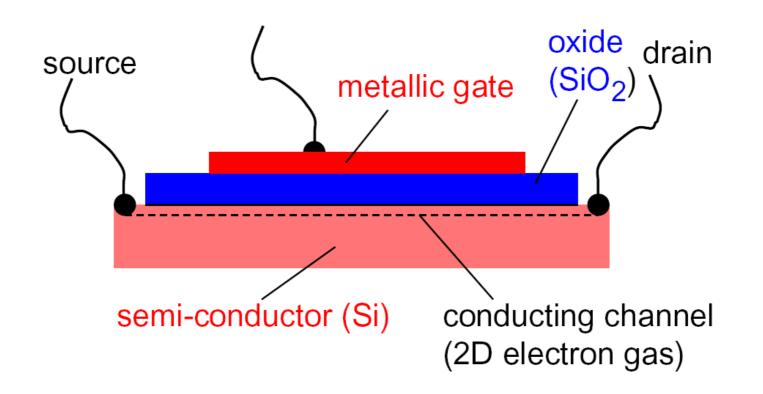
What is an electron?

electron = object with charge -e and spin $\frac{1}{2}$



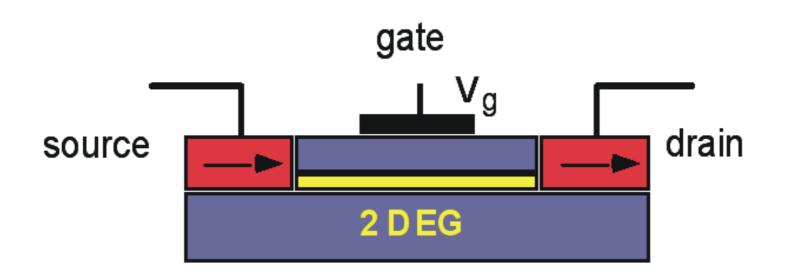
but so far, electronics has completely neglected the spin

"Normal" transistor (MOSFET)



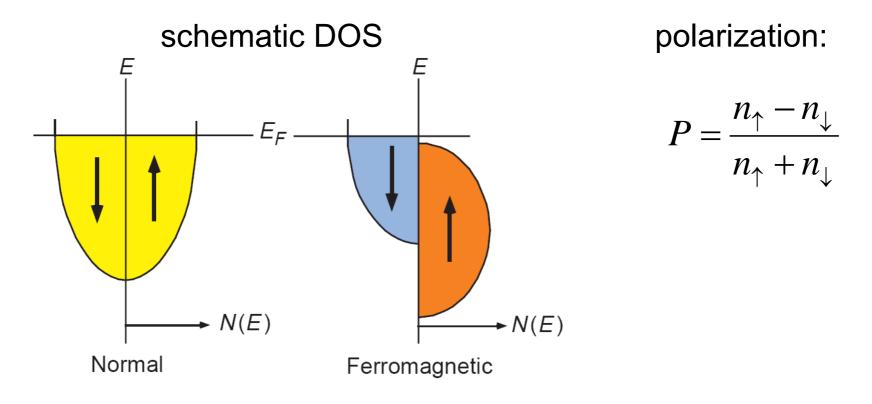
- gate voltage controls the current between source and drain
- used as modulator / amplifier or switch

Spin transistor



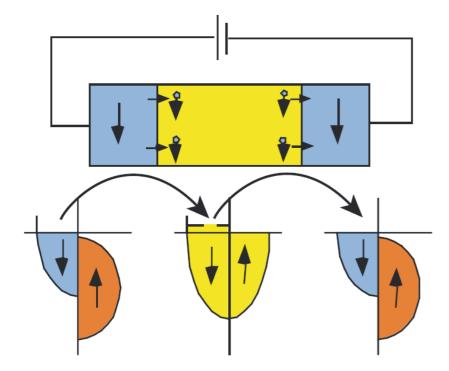
- source and drain are ferromagnetic materials
- gate voltage changes the spin of the electrons to control the current – no external magnetic field

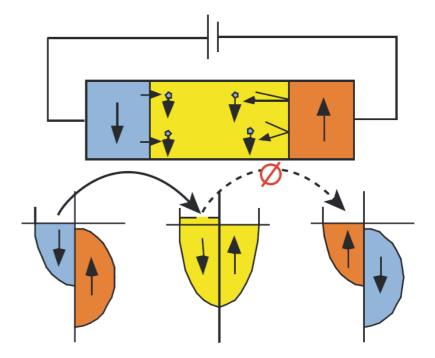
Spin-polarized transport



- imbalance of spin population at Fermi level leads naturally to spin-polarized transport
- commonly occurs in ferromagnetic metals (or alloys) with P up to 50 %

Spin valve





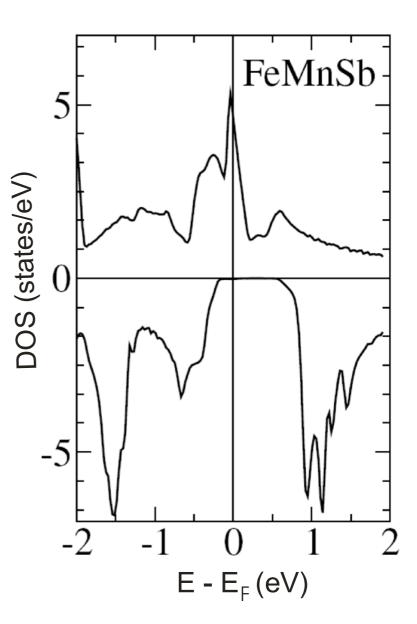
low resistance

high resistance

New materials

- Heusler alloys have a (theoretical) spin polarization of 100 %
- e.g. Co₂MnSi, FeMnSb

Mavropoulos *et al.* J. Phys.: Condens. Matter **16** (2004)



Spin-orbit coupling

• an electron moving in an electric field experiences a magnetic field in its rest frame of reference

$$\vec{B} \propto \vec{v} \times \vec{E}$$

 non-relativistic limit of Dirac equation leads to the Schrödinger equation with an additional term that describes spin-orbit coupling (SOC)

$$\hat{H}_{SOC} = \frac{\hbar}{(2mc)^2} \nabla V \cdot (\hat{\sigma} \times \hat{k})$$

Symmetry considerations

• from time reversal symmetry (Kramer's theorem)

$$E(\vec{k},\uparrow) = E(-\vec{k},\downarrow)$$

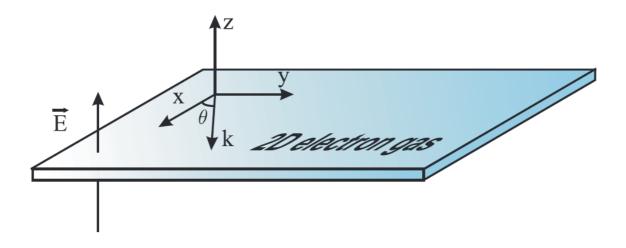
• from inversion symmetry

$$E(\vec{k},\uparrow) = E(-\vec{k},\uparrow)$$

 if both symmetries are fulfilled the bandstructure is doubly degenerated

$$E(\vec{k},\uparrow) = E(\vec{k},\downarrow)$$

quasi-2D free electron gas



on the surface of a solid or in heterostructures the inversion symmetry is broken and SOC can lift the degeneracy of the states

Rashba term

• spin-orbit Hamiltonian in 2D is so-called Rashba term

$$H_{SOC} = \alpha \left(\vec{\sigma} \times \vec{k} \right) \vec{e}_{z}$$
$$= \alpha \left(\sigma_{x} k_{y} - \sigma_{y} k_{x} \right)$$

coupling parameter α can be controlled by the electric field

$$\alpha \propto \nabla V$$

Ansatz

• full Hamiltonian of the 2DEG

$$H = \begin{pmatrix} \frac{1}{2m}(k_x^2 + k_y^2) & \alpha(k_y + ik_x) \\ \alpha(k_y - ik_x) & \frac{1}{2m}(k_x^2 + k_y^2) \end{pmatrix}$$

$$H\big|\psi\big\rangle = E\big|\psi\big\rangle$$

commutator

$$\left[k,H\right]_{-}=0$$

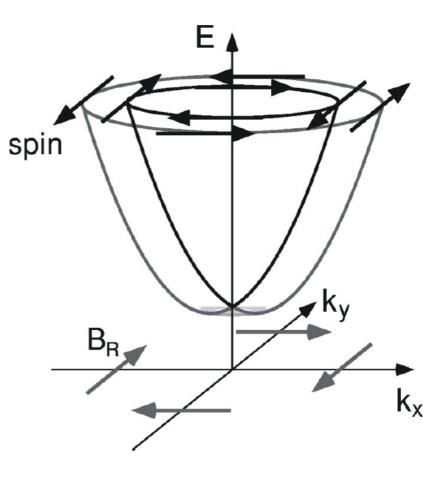
• wave function

$$|\psi\rangle = e^{i(k_x x + k_y y)} \left(a|\uparrow\rangle + b|\downarrow\rangle\right)$$

Eigenvalues and eigenstates

$$E_1(k,\uparrow) = \frac{k^2}{2m} + \alpha |k|$$
$$E_2(k,\downarrow) = \frac{k^2}{2m} - \alpha |k|$$

$$\psi_{1}(\vec{r}) = \frac{1}{\sqrt{2}} e^{i\vec{k}\vec{r}} \left(\left| \uparrow \right\rangle + ie^{i\Theta} \left| \downarrow \right\rangle \right)$$
$$\psi_{2}(\vec{r}) = \frac{1}{\sqrt{2}} e^{i\vec{k}\vec{r}} \left(ie^{-i\Theta} \left| \uparrow \right\rangle + \left| \downarrow \right\rangle \right)$$



Spin precession

 phase shift between two electrons with the same energy and different spin:

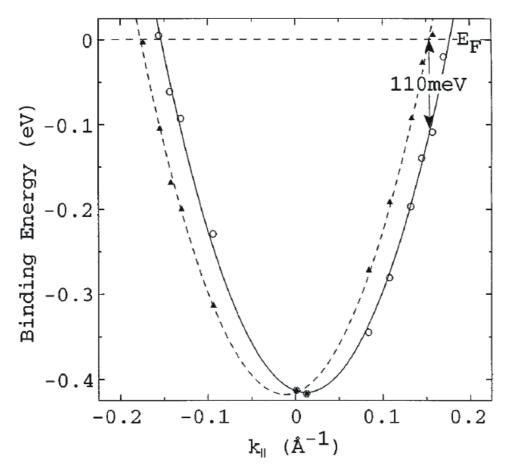
$$\Delta k = 2m\alpha$$
$$\Delta \theta = 2m\alpha L$$

 spin rotates analogue to Larmor precession with frequency

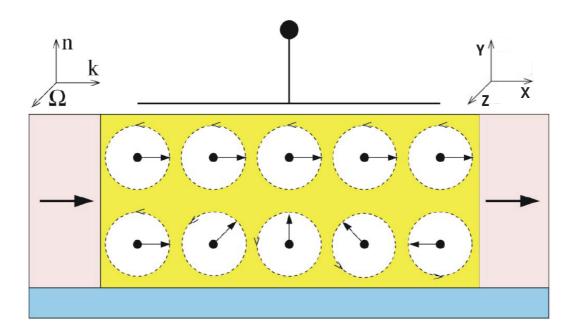
$$\Omega(\vec{k}) = \alpha(\vec{k} \times \vec{n})$$

Experimental results

spin splitting of Au(111) surface state LaShell *et al.* PRL **77** (1996)



Spin transistor revisited



x-polarized spin is a superposition of spin-up and spin-down eigenstates along the z-axis

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \longleftrightarrow \quad |x\rangle = |\uparrow\rangle + |\downarrow\rangle$$