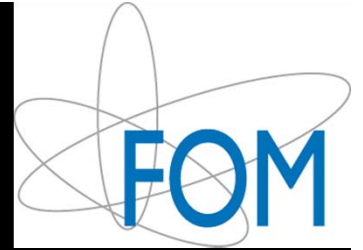




Radboud Universiteit Nijmegen



Lecture I

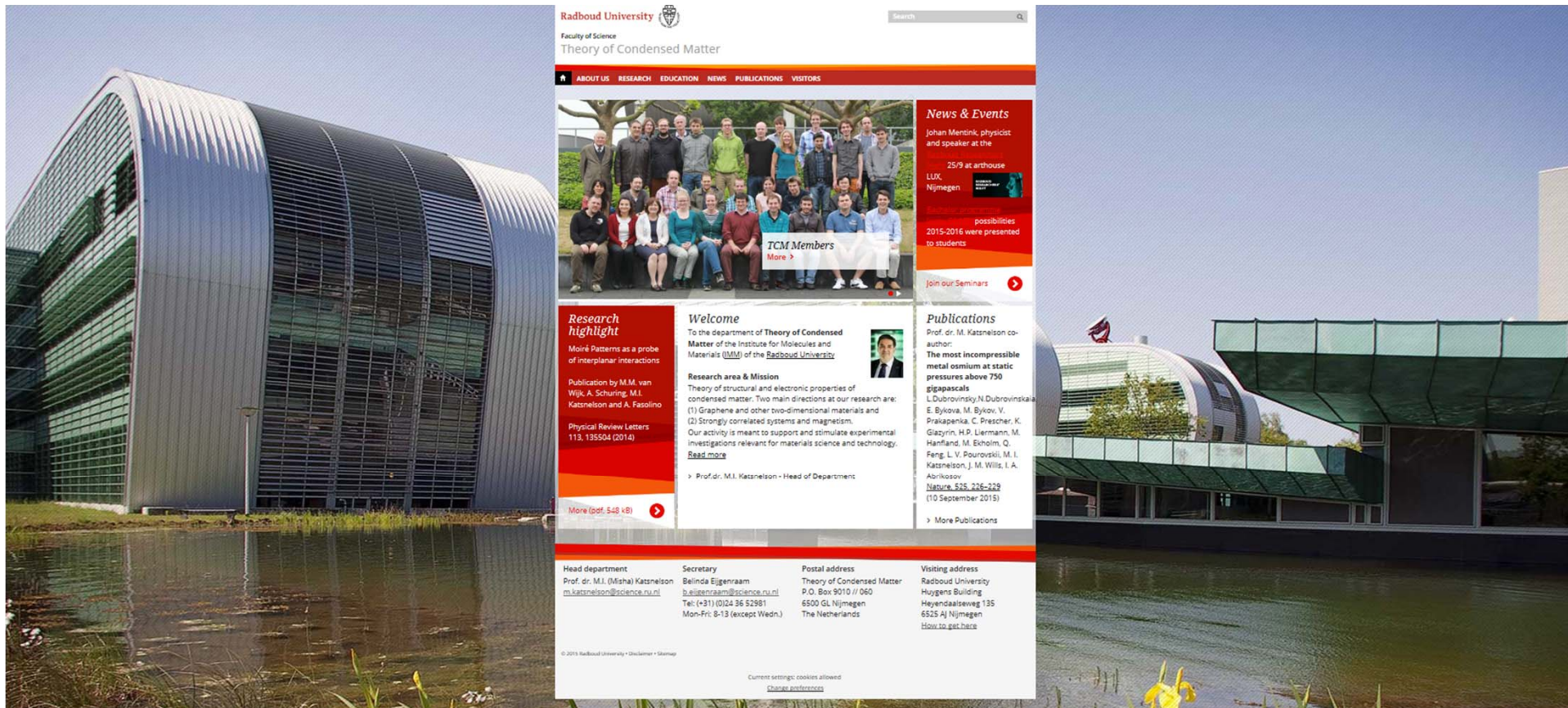
Spin-Orbitronics

Alireza Qaiumzadeh

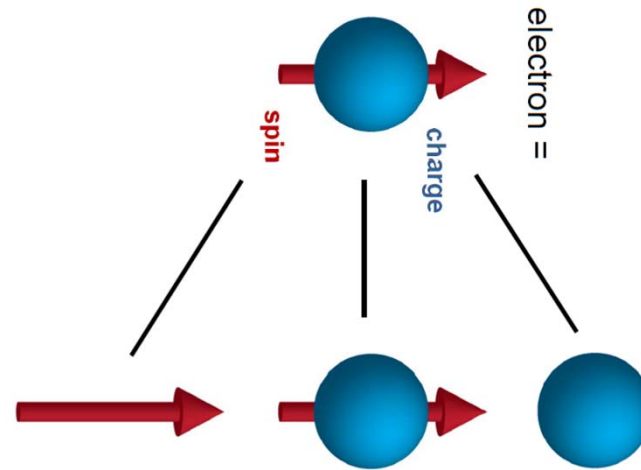
Radboud University (RU)

Institute for Molecules and Materials (IMM)

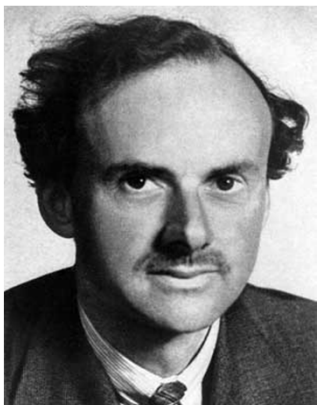
Theory of Condensed Matter group (TCM)



What We Talk About When We Talk About Spin-Orbitronics



“Mott” non-relativistic two-spin-channel model of ferromagnets



“Dirac” relativistic spin-orbit coupling

$$(\boldsymbol{\sigma} \times \nabla V) \cdot \mathbf{p}$$

What We Talk About When We Talk About Spin-Orbitronics

Anomalous Hall effect

Spin Hall effect and inverse effect

Spin galvanic effect and inverse effect

Spin-orbit torques and inverse effect

Magnon Hall effect

Topological insulators

⋮

Outlook:

1. Rashba spin-orbit interaction
2. Extraordinary Hall Effect
3. Spin transfer torques
4. Spin-orbit torques

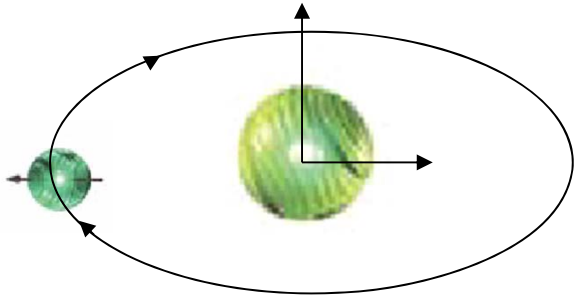
1. Rashba spin-orbit interaction

D. Bercioux and P. Lucignano, Rep. Prog. Phys. **78**, 106001 (2015).

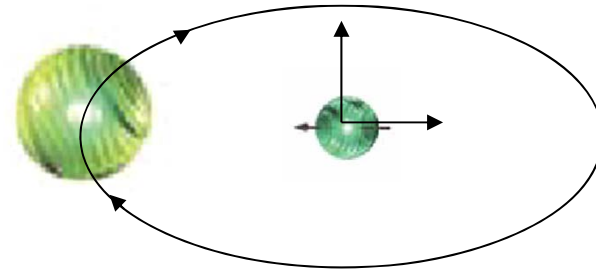
A. Manchon, H. C. Koo, J. Nitta, S. M. Frolov, and R. A. Duine, Nat. Mater. **14**, 871 (2015).

Spin-orbit coupling (classic description)

nucleus rest frame



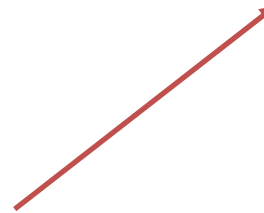
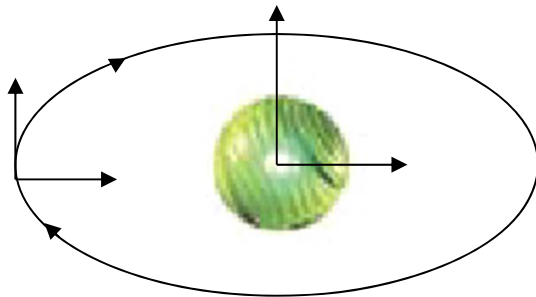
electron rest frame



$$\mathbf{I} = Q\mathbf{v} \quad \mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^3} \mathbf{r} \quad \mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{I} \times \mathbf{r}}{r^3}$$

$$\mathbf{B} = \mu_0 \epsilon_0 \mathbf{v} \times \mathbf{E} = \frac{1}{c^2} \mathbf{v} \times \mathbf{E} \quad H_{SO} = \frac{g\mu_B}{2\hbar} \mathbf{S} \cdot \mathbf{B} = \frac{e}{2mc^2} \mathbf{S} \cdot \mathbf{v} \times \mathbf{E}$$

Lorentz transformation → Thomas precession



Relativistic SOI

$$H_{\text{Dirac}} = c\boldsymbol{\alpha} \cdot (\mathbf{p} - e\mathbf{A}) + \beta mc^2 + eV,$$

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}$$

Pauli equation

$$H \sim \underbrace{\frac{1}{2m}(\mathbf{p} - e\mathbf{A})^2 + mc^2 + eV - \mu_B \boldsymbol{\sigma} \cdot \mathbf{B}} - \frac{e\hbar}{4m^2c^2} \boldsymbol{\sigma} \cdot [\mathbf{E} \times (\mathbf{p} - e\mathbf{A})] \\ - \frac{1}{8m^3c^2}(\mathbf{p} - e\mathbf{A})^4 + \frac{e\hbar^2}{8m^2c^2}(\nabla \cdot \mathbf{E})$$

$$H_{SO} = -\lambda_{vac} \boldsymbol{\sigma} \cdot (\mathbf{k} \times \nabla V)$$

$$\lambda_{vac} = \frac{e\hbar^2}{4m^2c^2} = 3.7 \times 10^{-6} A^2$$

$$H_{SO} = -\frac{e\hbar}{4m^2c^2} \boldsymbol{\sigma} \cdot (\mathbf{E} \times \mathbf{p}) = -\frac{e\hbar(-\frac{1}{r} \frac{dV}{dr}) \boldsymbol{\sigma} \cdot (\mathbf{r} \times \mathbf{p})}{4m^2c^2} \\ = \frac{e}{2m^2c^2} \frac{1}{r} \frac{dV}{dr} \mathbf{S} \cdot \mathbf{L}.$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

Intrinsic vs. Extrinsic SOI

$$\begin{aligned} V &= V_{cr} + \tilde{V} \\ H_{\text{int}} &= -\frac{1}{2}\mathbf{b}(\mathbf{k}) \cdot \boldsymbol{\sigma}, \\ H_{\text{ext}} &= -\lambda \boldsymbol{\sigma} \cdot (\mathbf{k} \times \nabla V_{\text{imp}}), \end{aligned}$$

Time reversal symmetry:

$$H_{\text{int}}(\mathbf{k}, \mathbf{r}, \boldsymbol{\sigma}) = H_{\text{int}}(-\mathbf{k}, \mathbf{r}, -\boldsymbol{\sigma}) \Rightarrow \mathbf{b}(\mathbf{k}) = -\mathbf{b}(-\mathbf{k})$$

Inversion symmetry:

$$H_{\text{int}}(\mathbf{k}, \mathbf{r}, \boldsymbol{\sigma}) = H_{\text{int}}(-\mathbf{k}, -\mathbf{r}, \boldsymbol{\sigma}) \Rightarrow \mathbf{b}(\mathbf{k}) = \mathbf{b}(-\mathbf{k})$$

$$\mathbf{b}(\mathbf{k}) = 0$$

Intrinsic Spin-Orbit Interaction

k-cubic Dresselhaus SOI:

n-doped 3D semiconductors with the bulk inversion asymmetry (BIA), III-V and II-VI zinc blende structures

$$H_{\text{int}}^{D-3d} = B[k_x(k_y^2 - k_z^2)\sigma_x + k_y(k_z^2 - k_x^2)\sigma_y + k_z(k_x^2 - k_y^2)\sigma_z]$$

k-linear Dresselhaus SOI:

the electrons are conned to 2D, in semiconductors with BIA

$$\begin{aligned} H_{\text{int}}^{\beta} &= \beta(k_x\sigma_x - k_y\sigma_y) \\ H_{\text{int}}^{D-2d} &= Bk_xk_y(k_y\sigma_x - k_x\sigma_y) \end{aligned} \quad \beta = -B(\pi/d)^2$$

K-linear Rashba SOI:

n-doped 3D semiconductors with the structure inversion asymmetry (SIA)

$$H_{\text{int}}^R = \alpha_R(k_y\sigma_x - k_x\sigma_y)$$

K-cubic Rashba SOI:

$$H_{\text{R-hole-2d}} = i\lambda_{\text{R-h}}(k_-^3\sigma_+ - k_+^3\sigma_-)$$

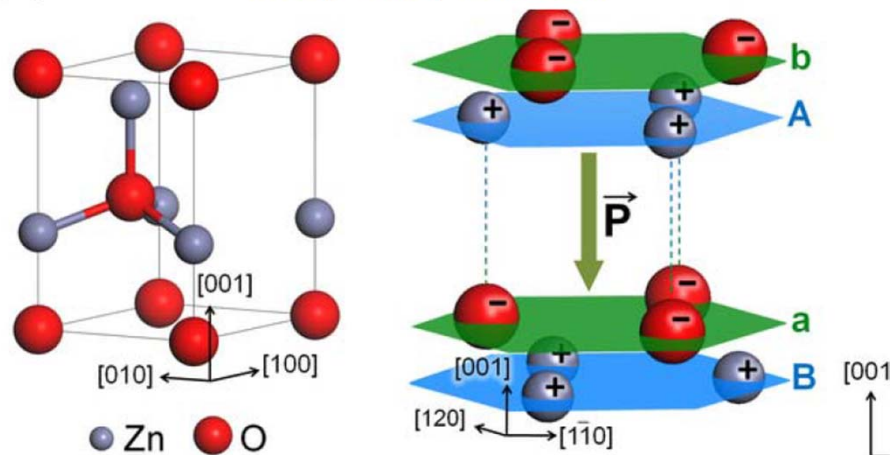
Luttinger SOI:

p-doped GaAs

$$H_{\text{KL}} = \frac{\hbar^2}{2m} \left[(2\gamma_1 + 5\gamma_2)k^2 - 2\gamma_3(\mathbf{k} \cdot \mathbf{J})^2 + 2(\gamma_3 - \gamma_2) \sum_i k_i^2 J_i^2 \right]$$

Rashba term – wurtzite structure

ZnO, GaN, CdSe, ...

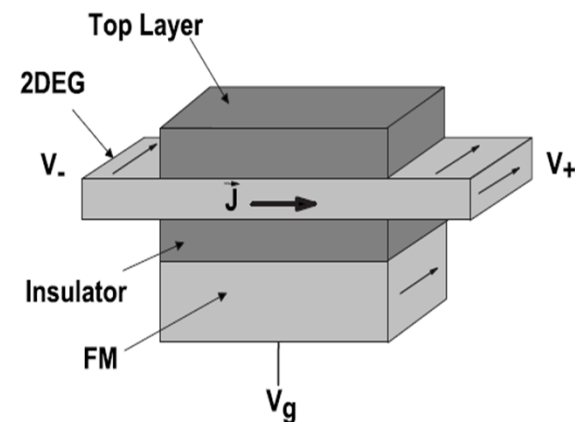
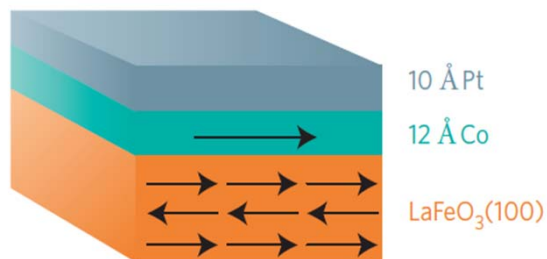
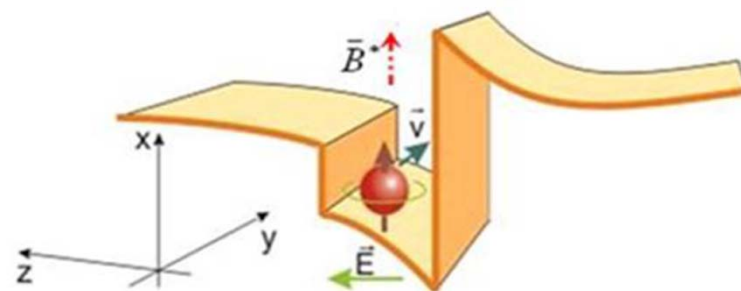


inversion symmetry broken and
time reversal symmetry conserved →

$$\mathcal{H}_R = \lambda_{so}(\mathbf{s} \times \mathbf{k}) \cdot \hat{\mathbf{c}}$$

E. I. Rashba, V. I. Sheka (FTT, 1959); B. C. Casella (IBM – PRL, 1960)

Rashba term – 2D Confinement



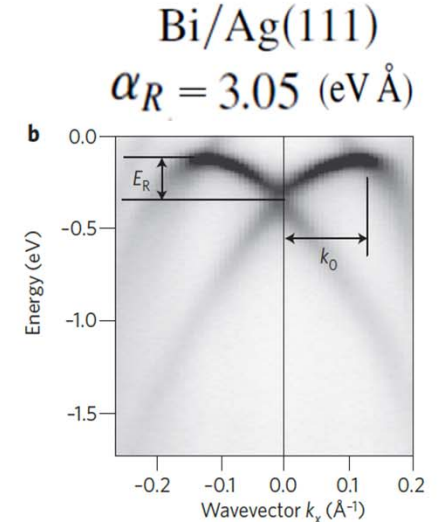
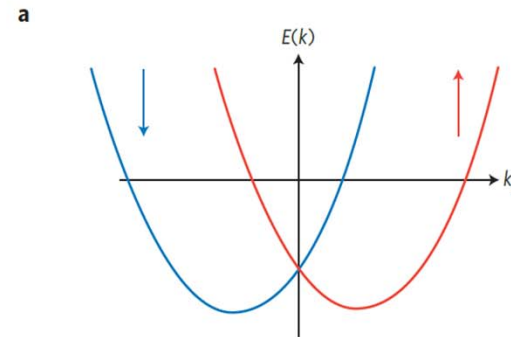
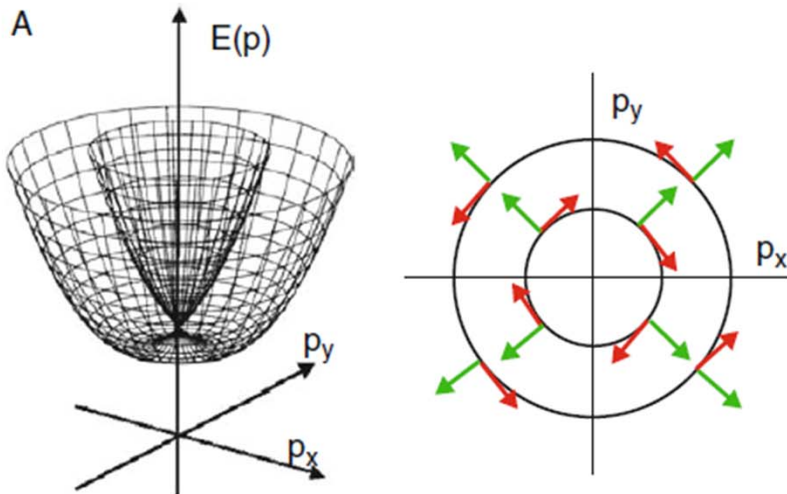
(Quasi) 2D systems with structure inversion asymmetry

$$\mathcal{H} = \frac{p^2}{2m_e} + \alpha_R (\boldsymbol{\sigma} \times \hat{z}) \cdot \mathbf{p}$$

$$E(k) = \frac{k^2}{2m} + s\alpha_R k$$

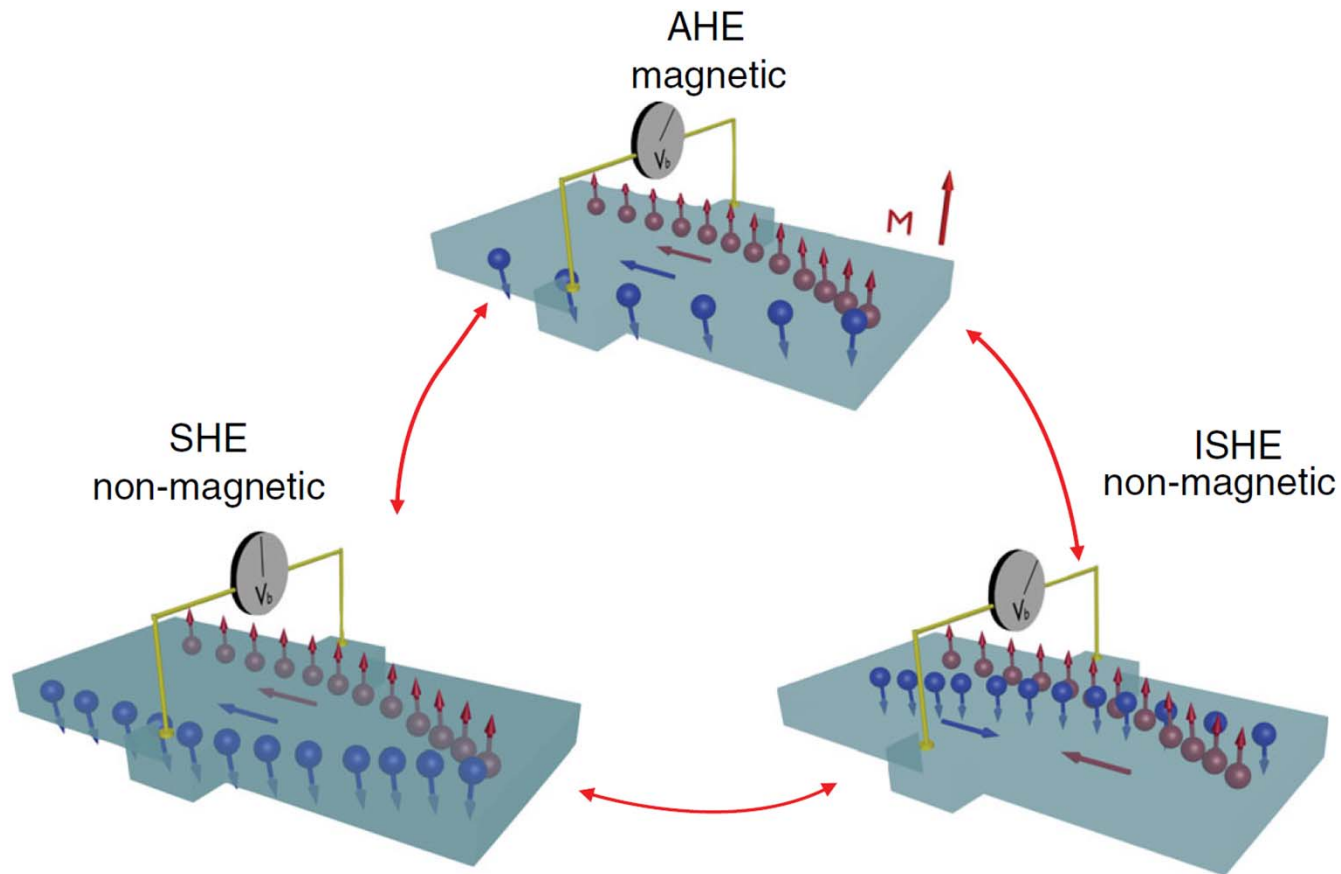
$$\psi_{\pm}(\mathbf{r}) = e^{i\mathbf{k} \cdot \mathbf{r}} \begin{pmatrix} \pm e^{i\phi} \\ 1 \end{pmatrix}.$$

1. E. Rashba, Properties of semiconductors with an extremum loop. 1. Cyclotron and combinational resonance in a magnetic field perpendicular to the plane of the loop. *Sov. Phys. Solid State* **2**, 1109–1122 (1960).
2. F. T. Vas'ko, Spin splitting in the spectrum of two-dimensional electrons due to the surface potential. *P. Zh. Eksp. Teor. Fiz.* **30**, 574–577 (1979).
3. Y. A. Bychkov & E. I. Rashba, Properties of a 2D electron gas with lifted spectral degeneracy. *P. Zh. Eksp. Teor. Fiz.* **39**, 66–69 (1984).



1. C. Ast, *et al.* Giant spin splitting through surface alloying. *Phys. Rev. Lett.* **98**, 186807 (2007).
2. A. Manchon, *et al.* New perspectives for Rashba spin-orbit coupling. *Nat. Mater.* **14**, 871 (2015).

2. Extraordinary Hall Effect



N. Nagaosa *et al.*, Rev. Mod. Phys. **82**, 1539 (2010).

D. Xiao *et al.*, Rev. Mod. Phys. **82**, 1959 (2010).

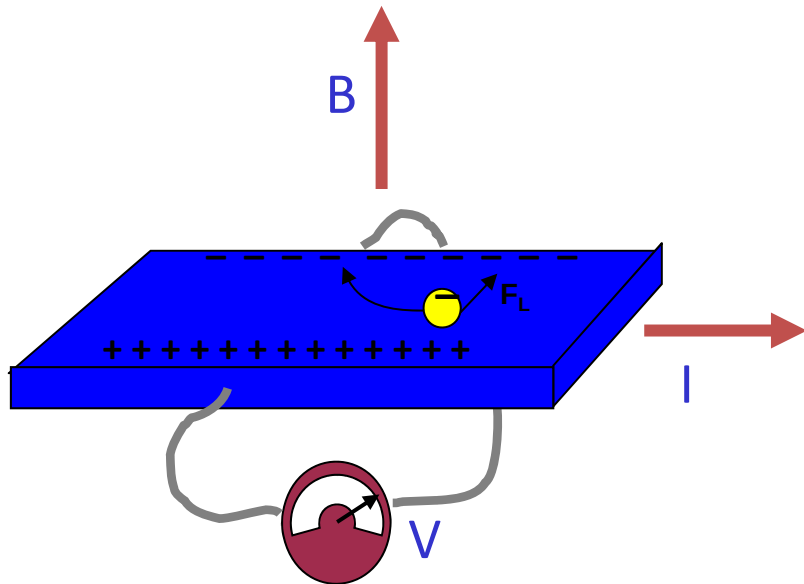
J. Sinova *et al.*, Rev. Mod. Phys. **87**, 1213 (2015).

Sinova and Jungwirth presentations

Extraordinary magnetoresistance

Queen of solid-state transport experiments

Ordinary magnetoresistance:
response to external magnetic field
Acting via classical Lorentz force
ordinary Hall effect (1879)



E. H. Hall, Amer. J. Math. **2**, 287 (1879)



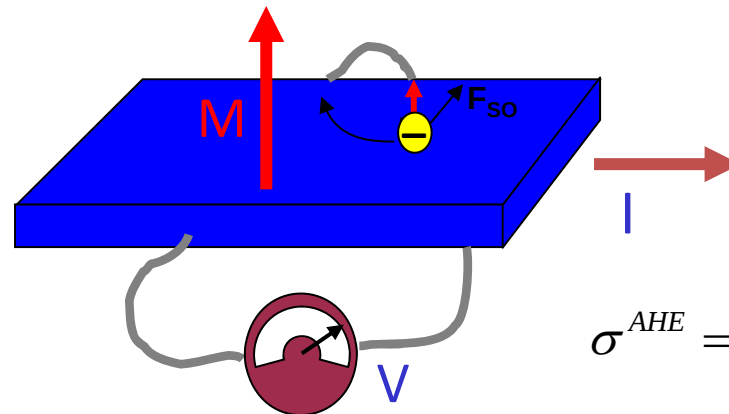
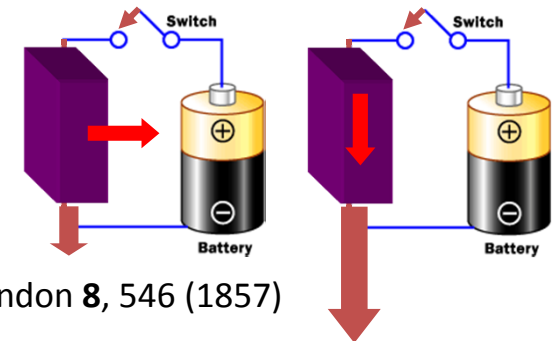
(Edwin Hall 1855-1938)

Extraordinary magnetoresistance:
response to internal quantum-relativistic
spin-orbit field

anisotropic magnetoresistance
(AMR) (1857)

$$\sigma^{AMR} = \sigma^s \equiv \frac{1}{2}(\sigma_{ij} + \sigma_{ji})$$

W. Thomson, Proc. Royal Soc. London **8**, 546 (1857)

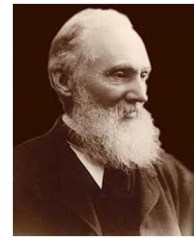


anomalous Hall effect
(AHE) (1881)

$$\sigma^{AHE} = \sigma^A \equiv \frac{1}{2}(\sigma_{ij} - \sigma_{ji})$$

$$\rho_{xy} = R_0 H_z + R_s M_z$$

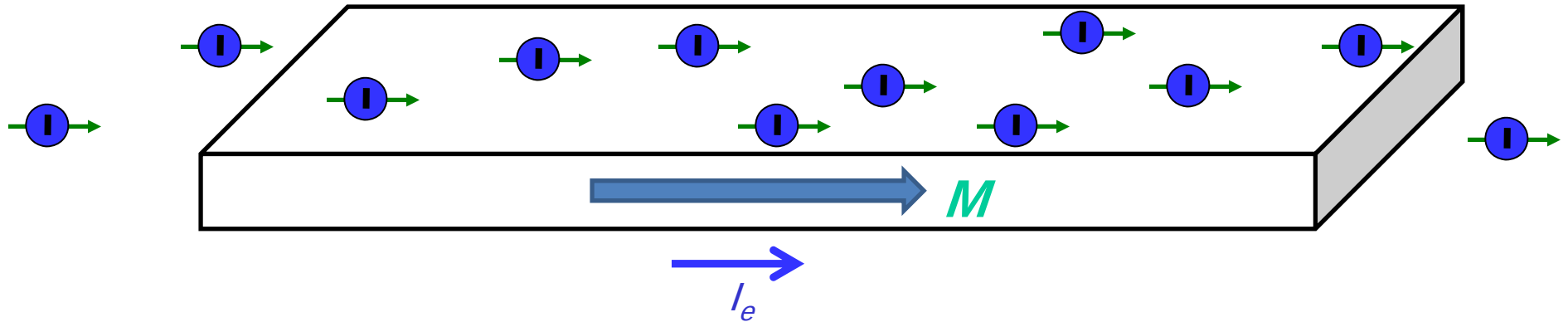
E. Hall, Phil. Mag. **12**, 157 (1881)



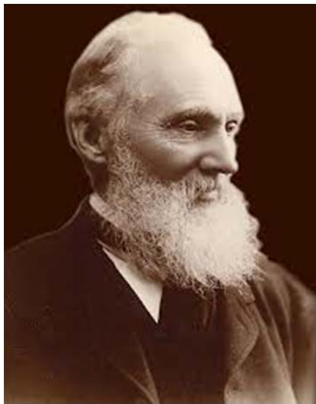
(Lord Kelvin
1824-1907)

Relativistic anisotropic magnetoresistance (AMR)

Spintronic effect 150 years ahead of time



XIX. "On the Electro-dynamic Qualities of Metals :—Effects of Magnetization on the Electric Conductivity of Nickel and of Iron." By Professor W. THOMSON, F.R.S. Received June 18, 1857.



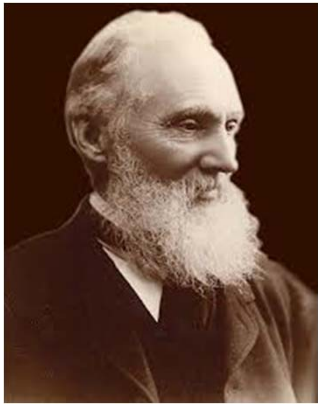
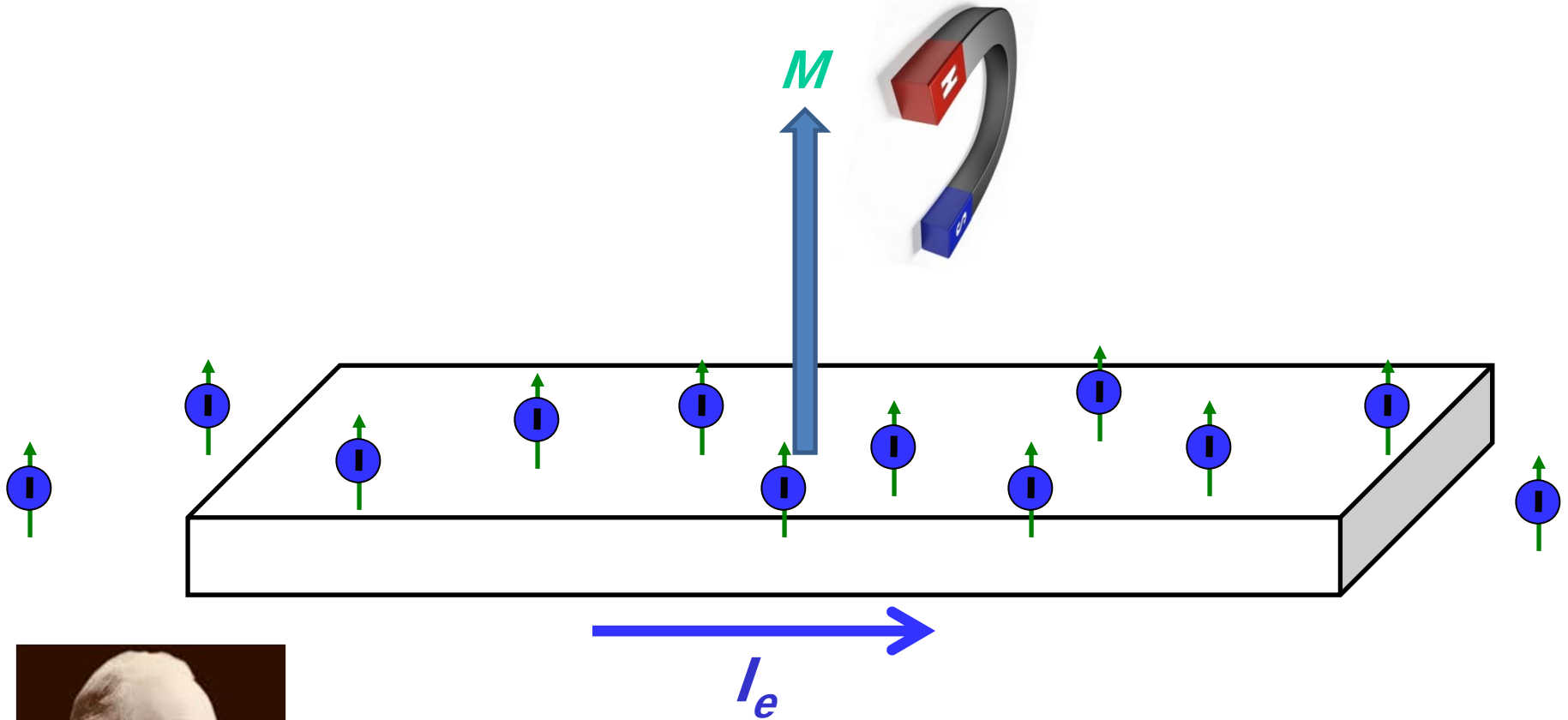
Kelvin, 1857

I have already communicated to the Royal Society a description of experiments by which I found that iron, when subjected to magnetic force, acquires an increase of resistance to the conduction of electricity along, and a diminution of resistance to the conduction of electricity across, the lines of magnetization*. By experiments more recently made, I have ascertained that the electric conductivity of nickel is similarly influenced by magnetism, but to a greater degree, and with a curious difference from iron in the relative magnitudes of the transverse and longitudinal effects.

W. Thomson, Proc. Royal Soc. London **8**, 546 (1857)

Read-out: relativistic anisotropic magnetoresistance (AMR)

Spintronic effect 150 years ahead of time



Kelvin, 1857

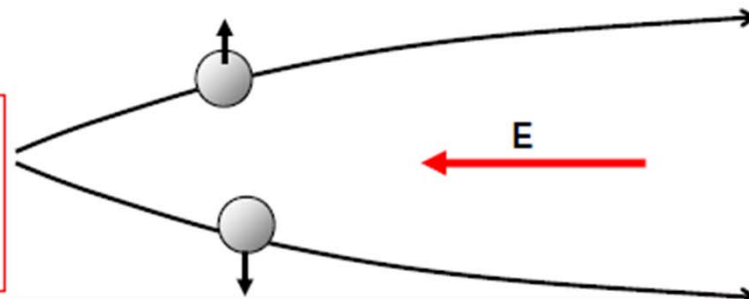
Mechanisms which are responsible for spin-orbitronics

a) Intrinsic deflection

Interband coherence induced by an external electric field gives rise to a velocity contribution perpendicular to the field direction. These currents do not sum to zero in ferromagnets.

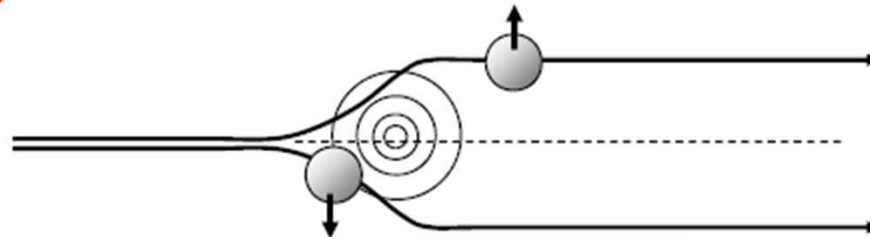
$$\frac{d\langle \vec{r} \rangle}{dt} = \frac{\partial E}{\hbar \partial \vec{k}} + \frac{e}{\hbar} \vec{E} \times \vec{b}_n$$

Electrons have an anomalous velocity perpendicular to the electric field related to their Berry's phase curvature



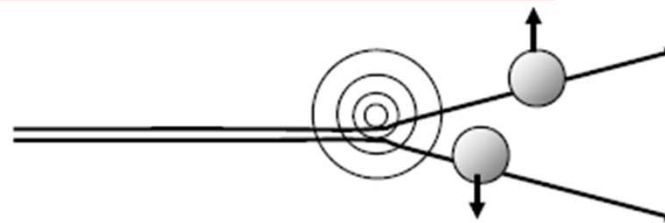
b) Side jump

The electron velocity is deflected in opposite directions by the opposite electric fields experienced upon approaching and leaving an impurity. The time-integrated velocity deflection is the side jump.

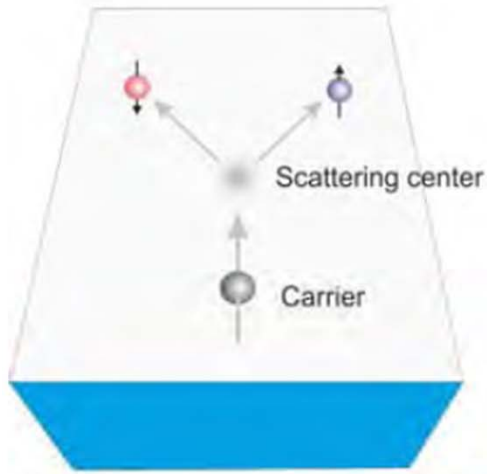


c) Skew scattering

Asymmetric scattering due to the effective spin-orbit coupling of the electron or the impurity.

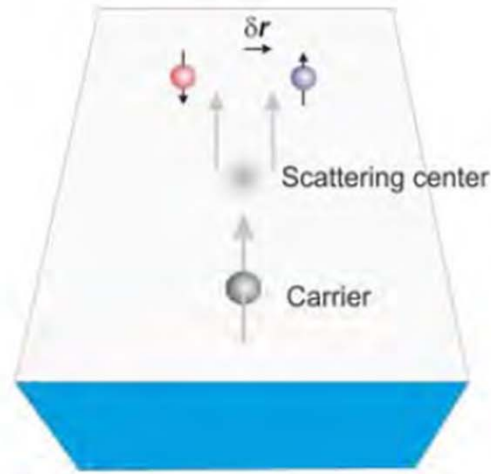


Extrinsic



Skew Scattering

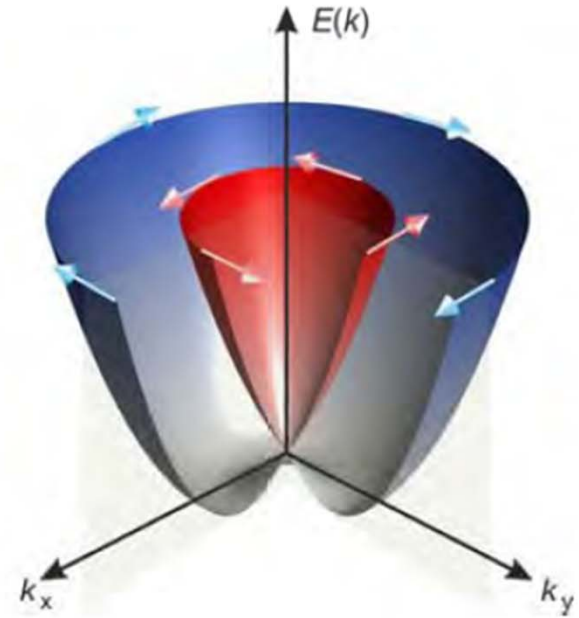
Smit, Physica 24, 39 (1958)



Side-Jump Scattering

Berger, PRB 2, 4559 (1970)

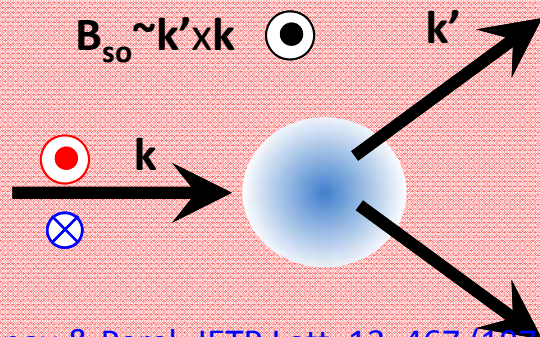
Intrinsic



Band Structure
E.g. Rashba

S. Zhang, PRL **85**, 393 (2000); S. Murakami, N. Nagaosa, S.C. & Zhang. Science **301**, 1348 (2003); J. Sinova, *et al.*, PRL **92**, 126603 (2004).

Mott scattering

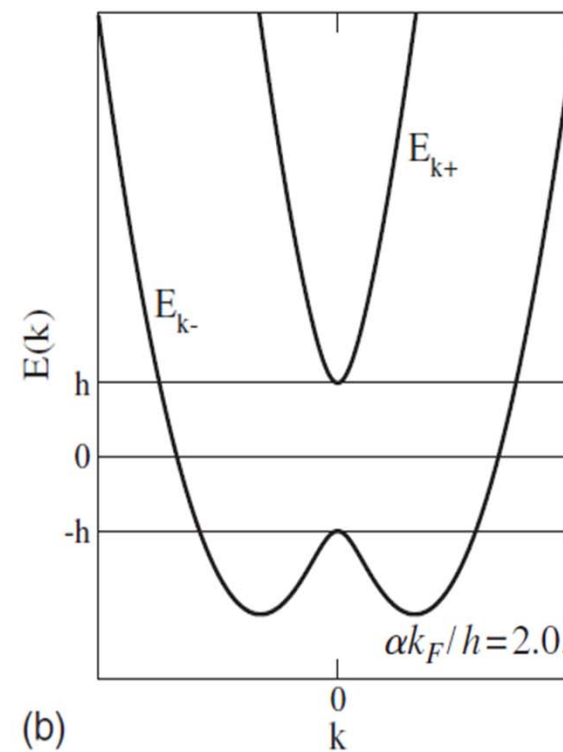
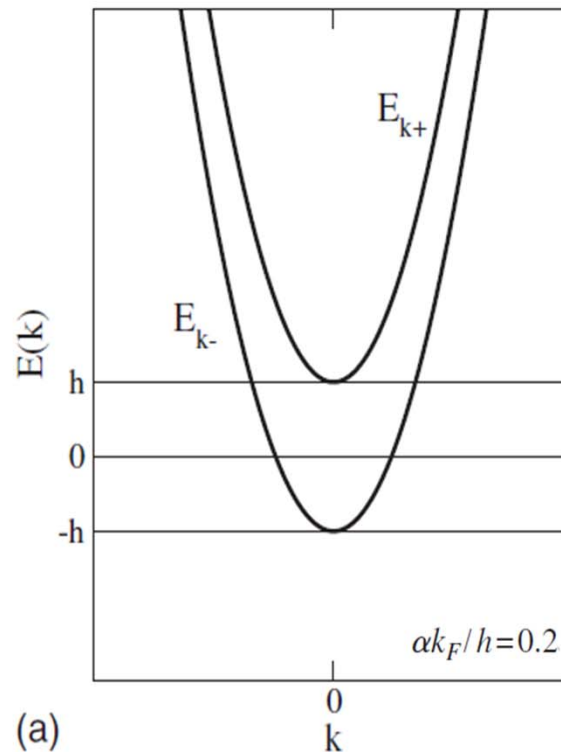


D'yakonov & Perel, JETP Lett. 13, 467 (1971)

Intrinsic Anomalous Hall effect

In systems with broken time-reversal symmetry and SOI

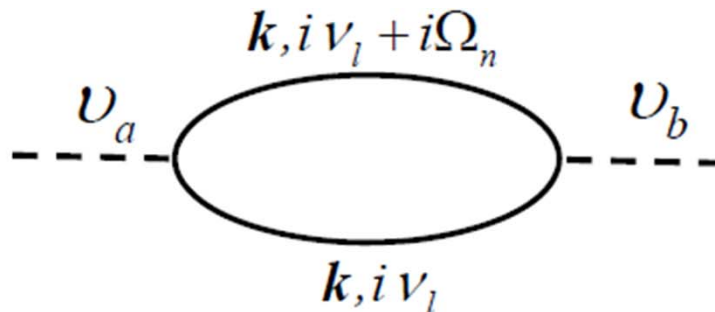
$$H = \frac{k^2}{2m}\sigma_0 + \alpha(\sigma_x k_y - \sigma_y k_x) - h\sigma_z + V(\mathbf{r})\sigma_0$$



Nunner *et al.* Phys. Rev. B **76**, 235312 (2007)

Sinitsyn *et al.* Phys. Rev. B **75**, 045315 (2007)

Intrinsic Anomalous Hall effect



$$\sigma_{ab}(\Omega) = \frac{ie^2}{\Omega} \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{1}{\beta} \sum_l \text{Tr} \{ \langle v_a G_0(\mathbf{k}, i\nu_l) v_b G_0(\mathbf{k}, i\nu_l + i\Omega_n) \rangle \}_{i\Omega_n \rightarrow \Omega + i0},$$

$$v_a = \frac{\partial \mathcal{H}_0}{\partial k_a} \longrightarrow v_x = \frac{k_x}{m} \sigma_0 - \alpha \sigma_y, \quad v_y = \frac{k_y}{m} \sigma_0 + \alpha \sigma_x,$$

$$\sigma_{yx} = \sigma_{yx}^{\text{I(a)}} + \sigma_{yx}^{\text{I(b)}} + \sigma_{yx}^{\text{II}},$$

$$\sigma_{yx}^{\text{I(a)}} = \frac{e^2}{2\pi V} \text{Tr} \langle v_y G^R(\epsilon_F) v_x G^A(\epsilon_F) \rangle,$$

$$\sigma_{yx}^{\text{I(b)}} = -\frac{e^2}{4\pi V} \text{Tr} \langle v_y G^R(\epsilon_F) v_x G^R(\epsilon_F) + v_y G^A(\epsilon_F) v_x G^A(\epsilon_F) \rangle,$$

$$\sigma_{yx}^{\text{II}} = \frac{e^2}{4\pi V} \int_{-\infty}^{\infty} d\epsilon f(\epsilon) \text{Tr} \left\langle v_y G^R(\epsilon) v_x \frac{\partial G^R(\epsilon)}{\partial \epsilon} - v_y \frac{\partial G^R(\epsilon)}{\partial \epsilon} v_x G^R(\epsilon) - v_y G^A(\epsilon) v_x \frac{\partial G^A(\epsilon)}{\partial \epsilon} + v_y \frac{\partial G^A(\epsilon)}{\partial \epsilon} v_x G^A(\epsilon) \right\rangle.$$

$$\sigma_{yx}^{\Pi}=-\frac{e^2}{4\pi}\bigg(1-\frac{h}{\sqrt{h^2+2\alpha^2m\epsilon_F+(\alpha^2m)^2}}\bigg)\Theta(h-\epsilon_F)$$

$$\sigma_{yx}^{\mathrm{I}(a),b}=-\frac{e^2}{2\pi}\frac{\alpha^2mh}{\lambda_F^2}$$

$$\lambda_F\!=\!\sqrt{h^2\!+\!2\alpha^2m\epsilon_F}$$

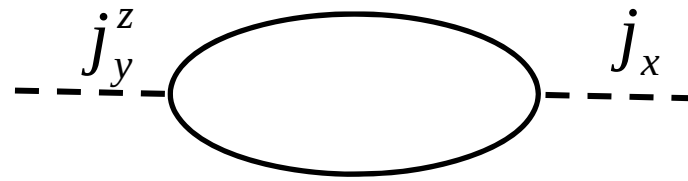
$$-\hspace{0.1cm} \overset{\Gamma_a}{\text{---}} \text{---} \triangleleft \hspace{0.1cm} = \hspace{0.1cm} -\hspace{0.1cm} \overset{\nu_a}{\text{---}} \text{---} \text{---} \hspace{0.1cm} + \hspace{0.1cm} -\hspace{0.1cm} \overset{\Gamma_a}{\text{---}} \text{---} \triangleleft \hspace{0.1cm} \times$$

$$n_iV_0^2\int\int\frac{dkkd\phi}{(2\pi)^2}G^R(\epsilon_F)v_xG^A(\epsilon_F)=\gamma_x\sigma_x+\gamma_y\sigma_y$$

$$\sigma_{yx}^{\mathrm{I}(a),b}+\sigma_{yx}^{\mathrm{I}(a),l}=0$$

Intrinsic Spin Hall Effect

In systems with time-reversal symmetry and SOI:



$$\mathbf{J}^z = (\hbar/4)\{\sigma^z, \mathbf{v}\}$$

$$\sigma_{sH}^0 = \frac{e}{8\pi} \left(1 - \frac{1}{1 + (\Delta\tau)^2}\right) \xrightarrow{\text{Disorder free limit}} \sigma_{sH} = \frac{e}{8\pi}$$

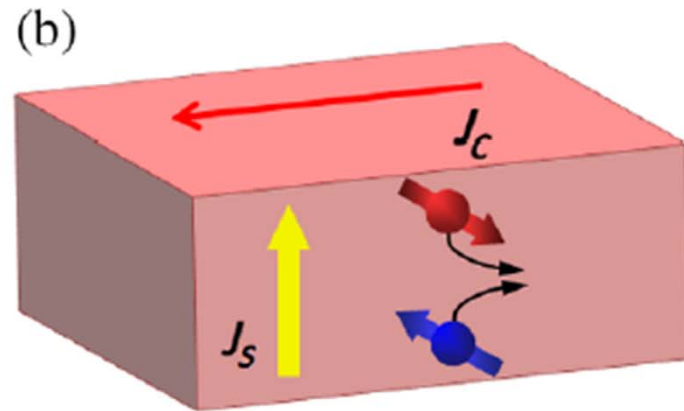
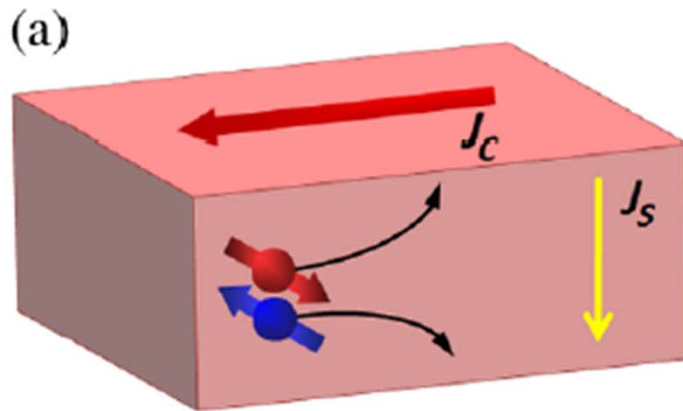
$$\Delta = 2p_F\alpha$$

$$\tilde{J}_y^z = \text{[triangle diagram]} + \text{[triangle diagram]} + \dots$$

(Inverse) Spin Hall Effect

$$\vec{J}_S = \theta_{SH}(\hbar/2e)\vec{J}_C \times \vec{\sigma}$$

$$\vec{J}_C = \theta_{SH}(2e/\hbar)\vec{J}_S \times \vec{\sigma}$$

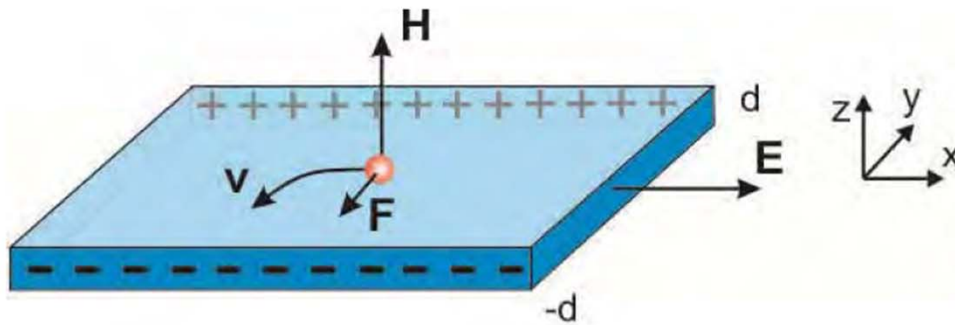


$$\sigma_{SH} = [\hbar/(2e)]\theta_{SH}\sigma$$

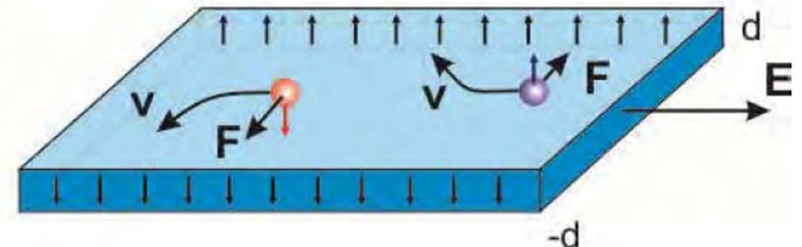
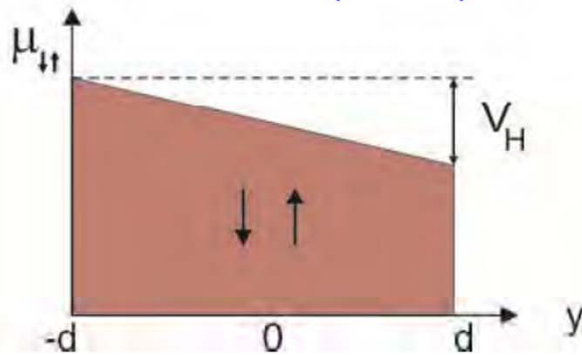
Magnus effect: <https://www.youtube.com/watch?v=2OSrvzNW9FE>

B. F. Miao, S.Y. Huang, D. Qu, and C. L. Chien, PRL **111**, 066602 (2013)

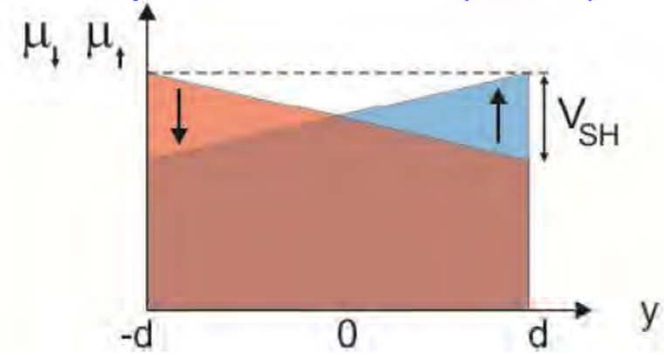
The spin Hall effect has the symmetry of the conventional Hall effect



Hall effect (1879)



Spin Hall effect (1971)



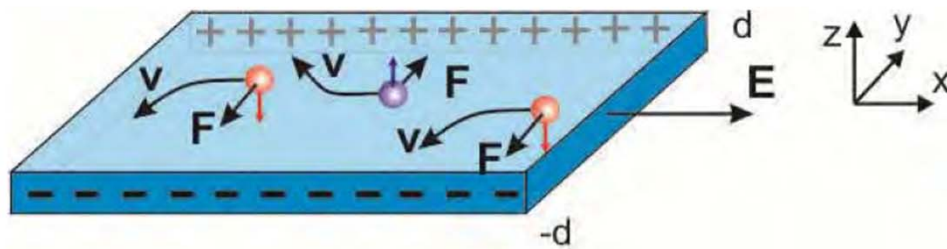
M.I. Dyakonov & V.I. Perel, JETP Lett. **13**, 467 (1971); J.E. Hirsch, PRL **83**, 1834 (1999);

S. Zhang, PRL **85**, 393 (2000); S. Murakami, N. Nagaosa, S.C. & Zhang. Science **301**, 1348 (2003);

J. Sinova, *et al.*, PRL **92**, 126603 (2004).

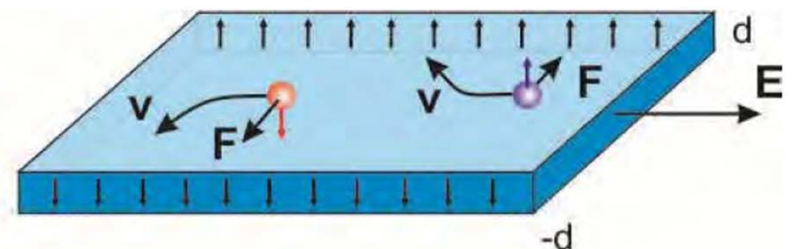
Scattering of unpolarized electrons by an unpolarized target results in spatial separation of electrons with different spins due to spin-orbit interaction

N. F. Mott and H. S. W. Massey, *The theory of atomic collisions* (Clarendon Press, Oxford, 1965)



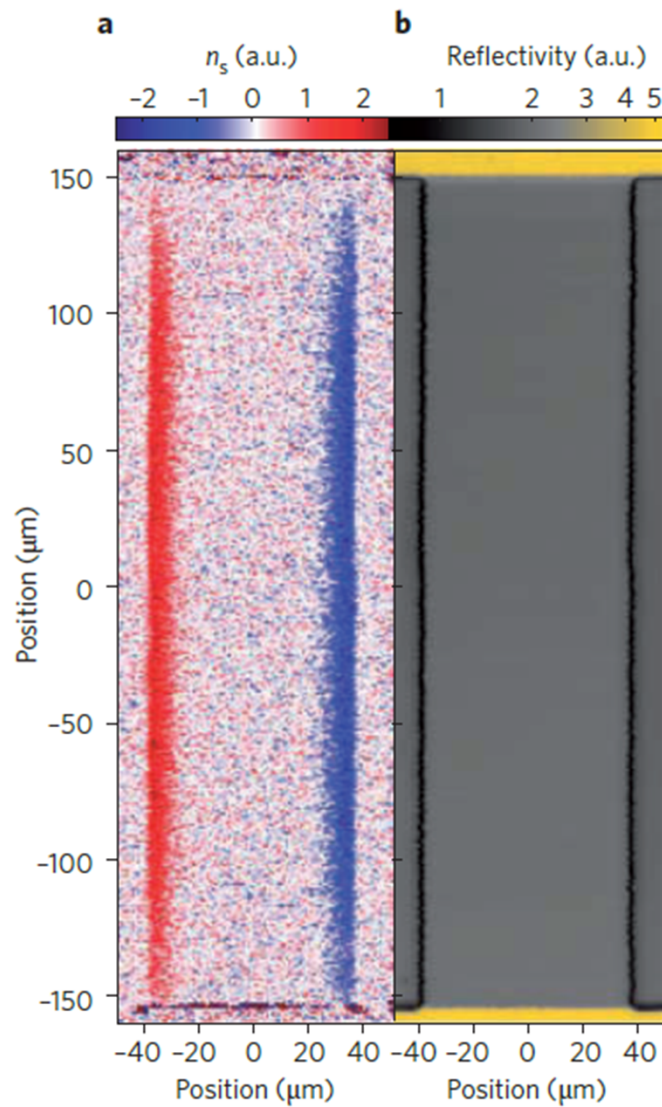
Anomalous Hall effect (1881)

E.H. Hall, *Phil . Mag.* 12, 157 (1881)



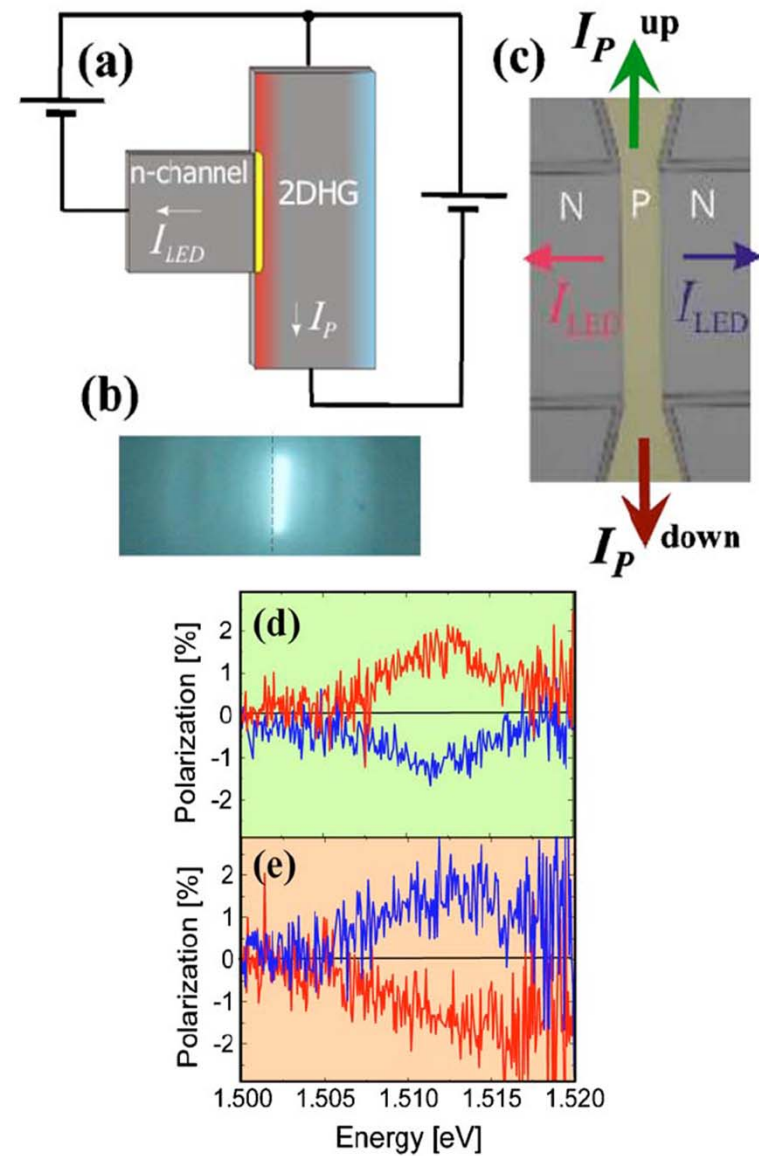
Spin Hall effect

Magneto-optical Kerr microscopy



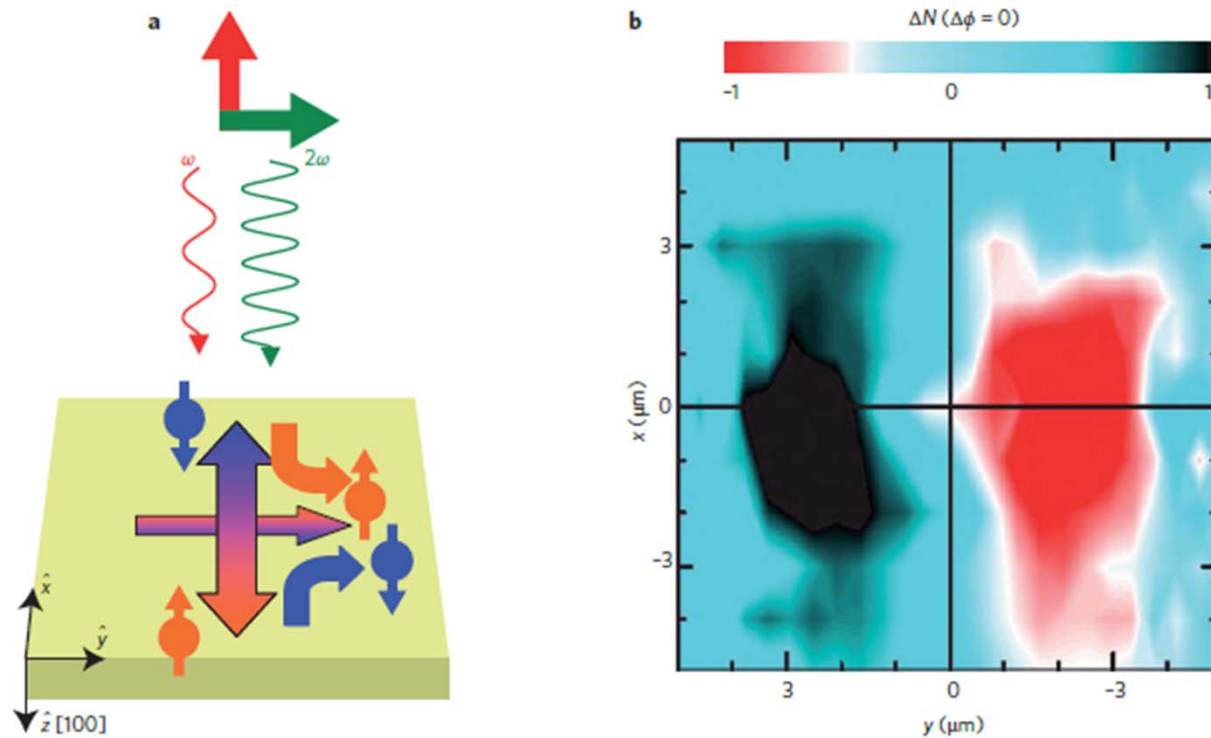
Extrinsic SHE *Kato, Awschalom, et al., Science'04*

Edge polarized electro-luminescence



Intrinsic SHE *Wunderlich, Kaestner, Sinova, TJ, PRL'05*

Inverse Spin Hall Effect



Zhao, H. *et al. Phys. Rev. Lett.* **96**, 246601 (2006).