Mercury Telluride
a
Topological Insulator

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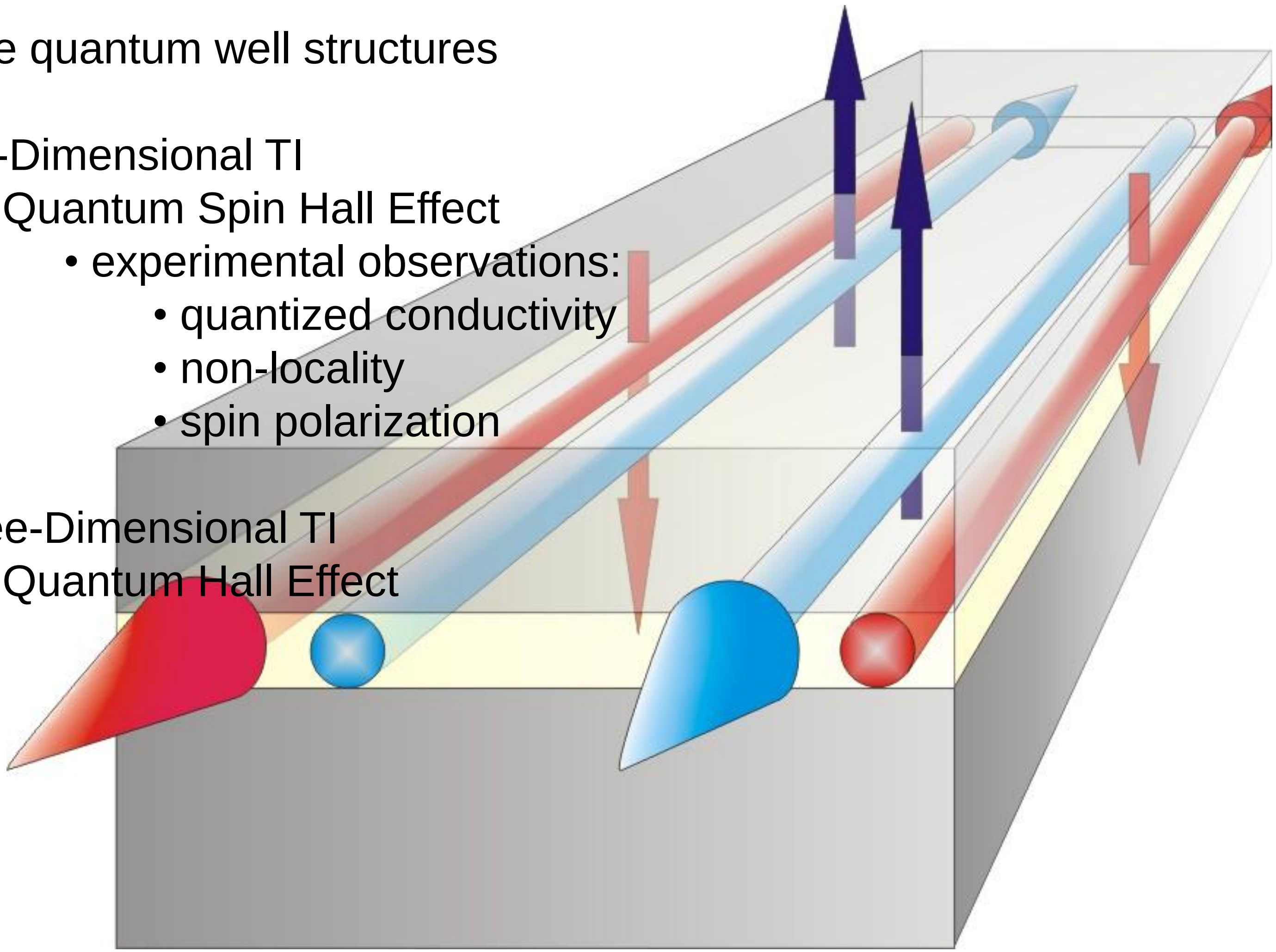
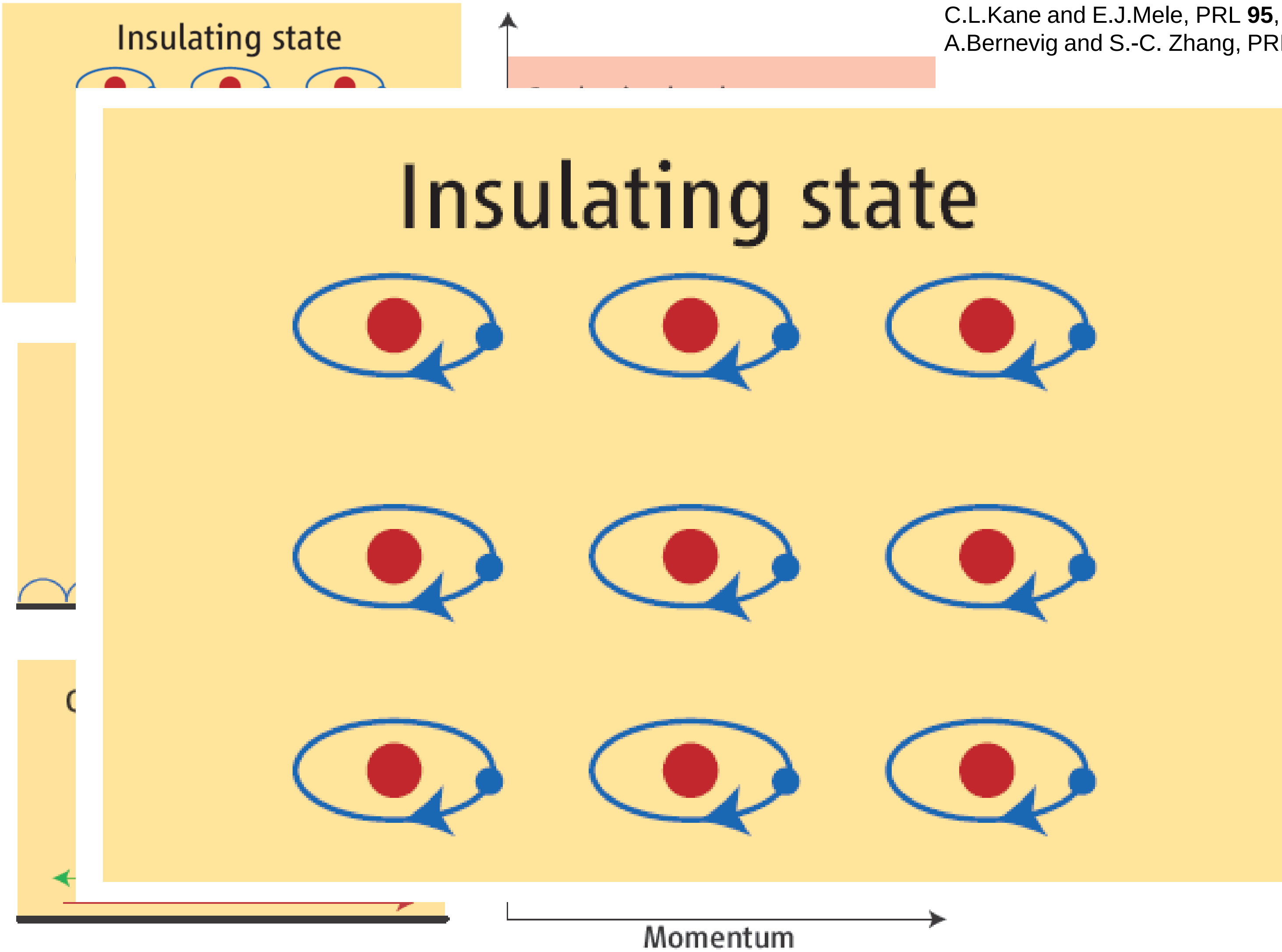
Outline

- Insulators and Topological Insulators
- HgTe quantum well structures
- Two-Dimensional TI
    - Quantum Spin Hall Effect
        - experimental observations:
            - quantized conductivity
            - non-locality
            - spin polarization
- Three-Dimensional TI
    - Quantum Hall Effect



The Insulating States Topologically Generalized

C.L.Kane and E.J.Mele, PRL **95**, 146802 (2005)
C.L.Kane and E.J.Mele, PRL **95**, 226801 (2005)
A.Bernevig and S.-C. Zhang, PRL **96**, 106802 (2006)

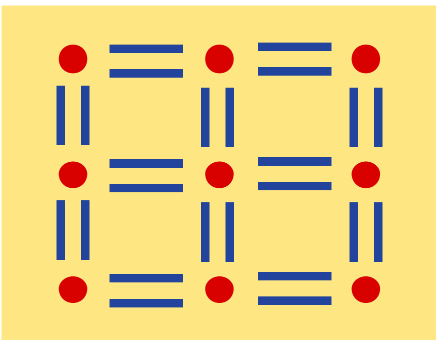


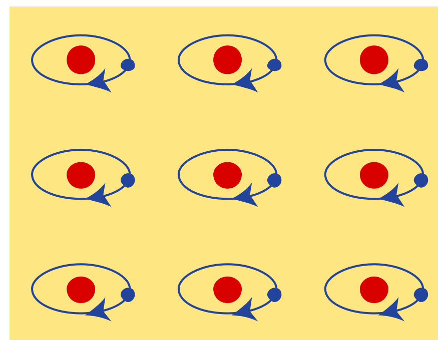
C.L.Kane and E.J. Mele, Science **314,** 1692 (2006)

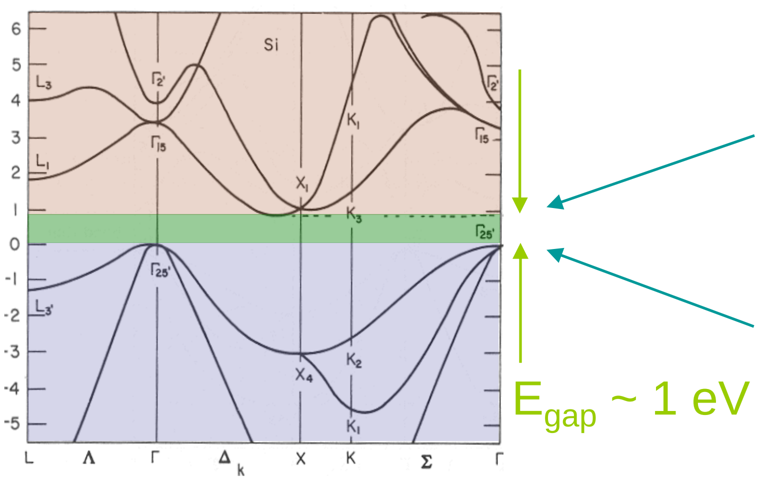
The Usual Boring Insulating State

Characterized by energy gap: absence of low energy electronic excitations

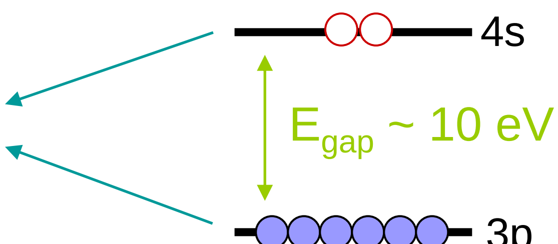
Covalent Insulator	Atomic Insulator	The vacuum
e.g. intrinsic semiconductor	e.g. solid Ar	

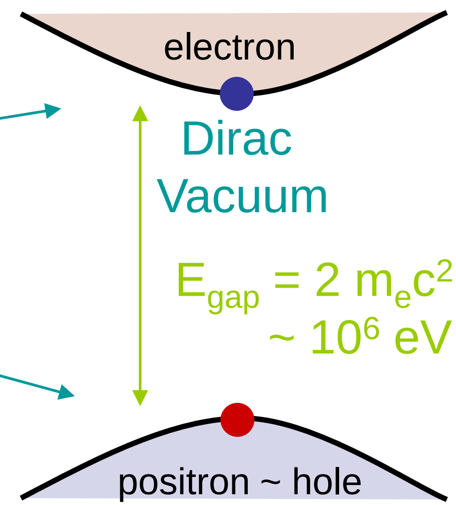






Silicon





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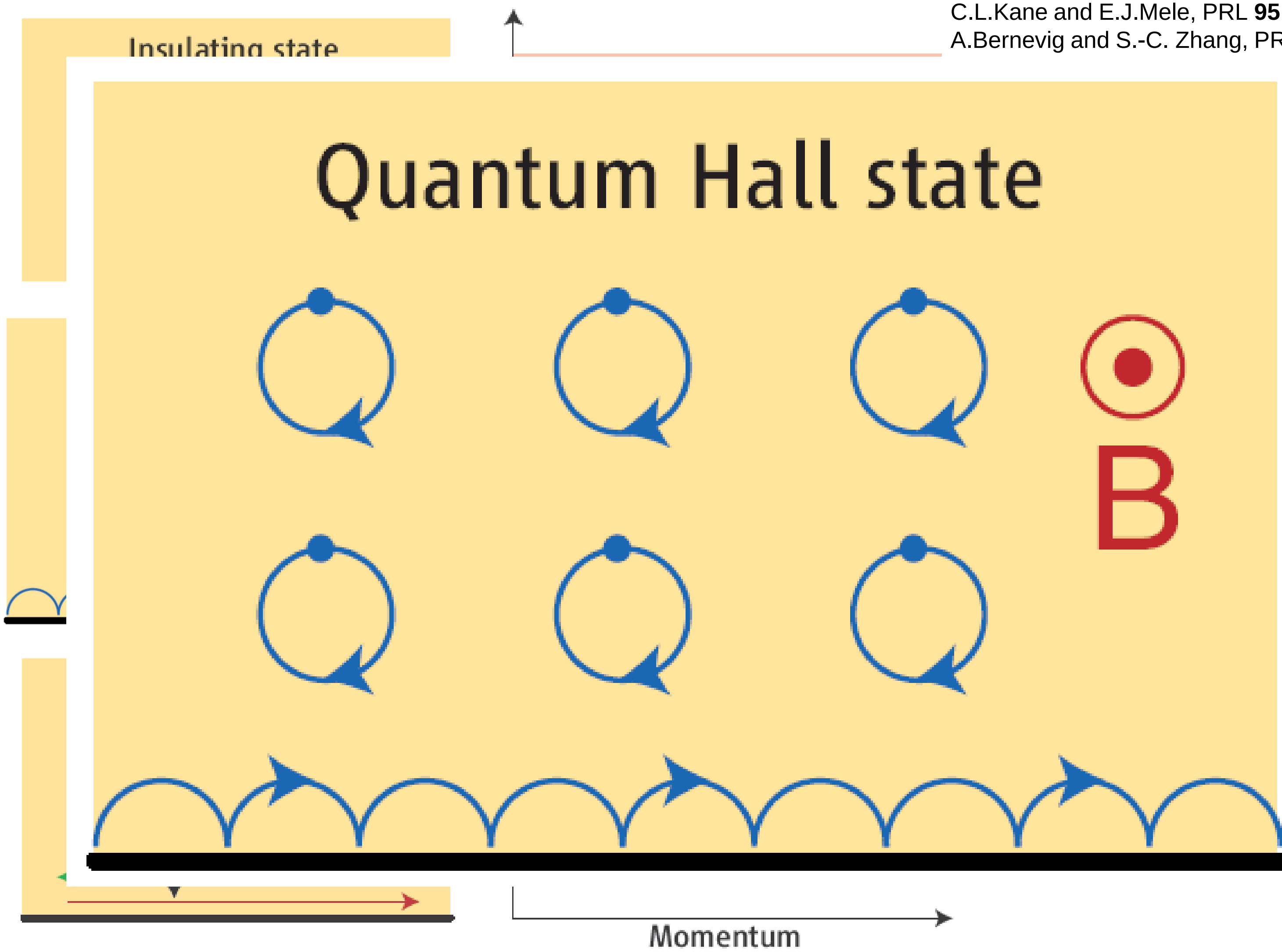
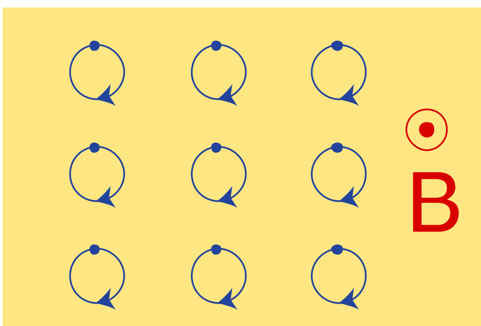
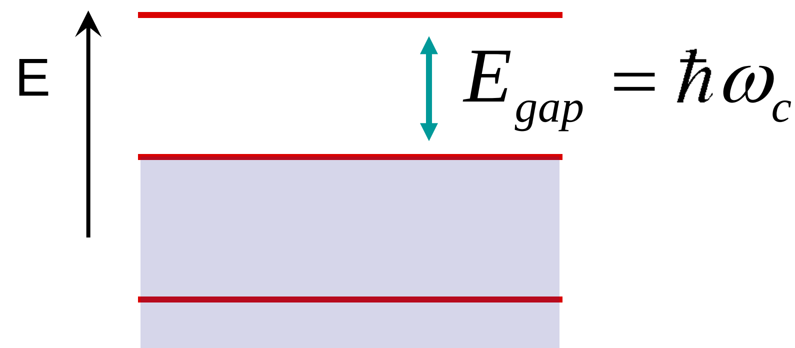


C.L.Kane and E.J. Mele, Science **314,** 1692 (2006)

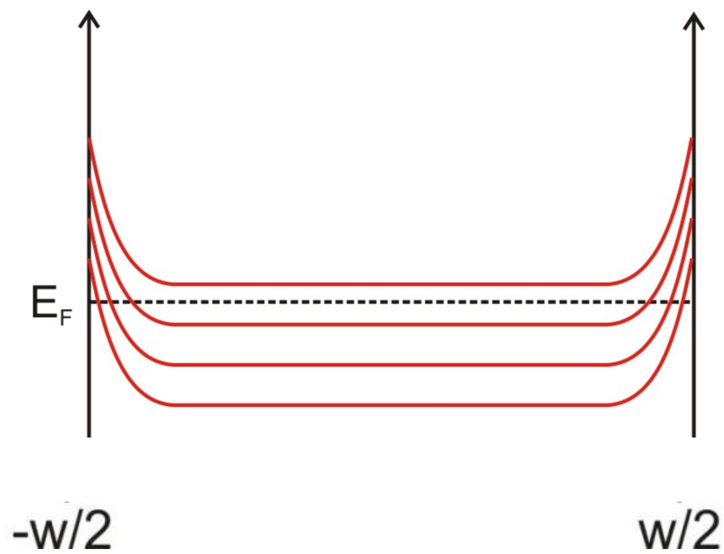
2D Cyclotron Motion, Landau Levels

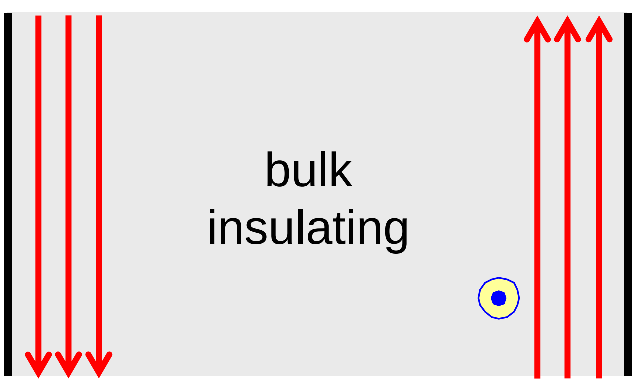


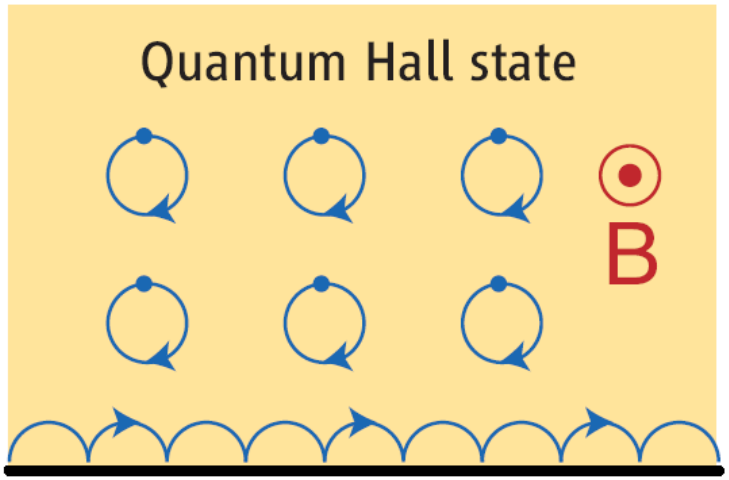


Energy gap, but NOT an insulator

Edge States







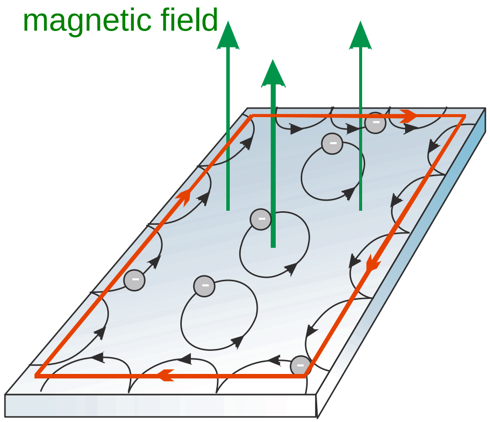
Quantized Hall conductivity :

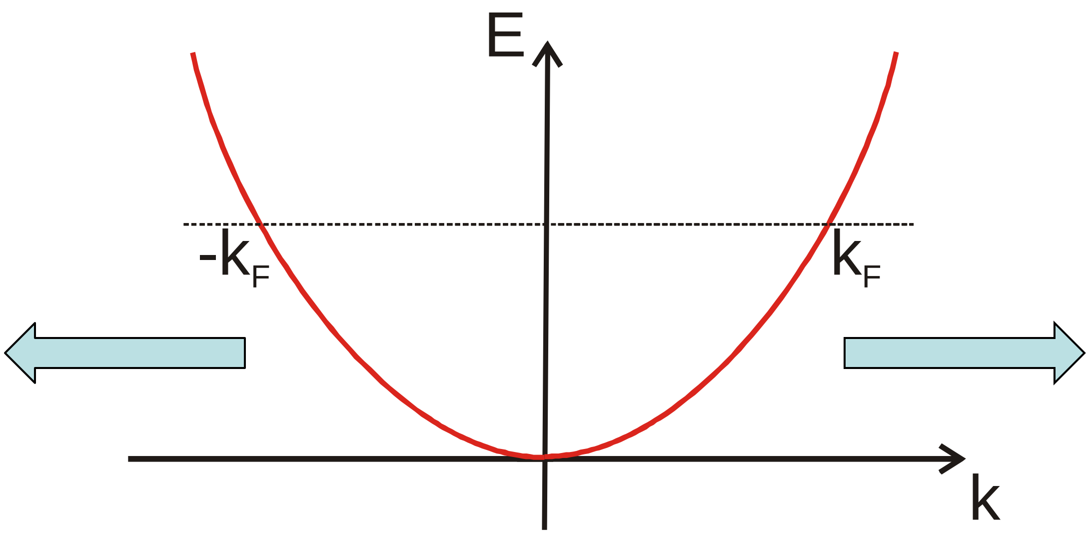
$$J_y = \sigma_{xy} E_x$$

$$\sigma_{xy} = n \frac{e^2}{h}$$

Integer accurate to 10^{-9}

Perfect Conductor



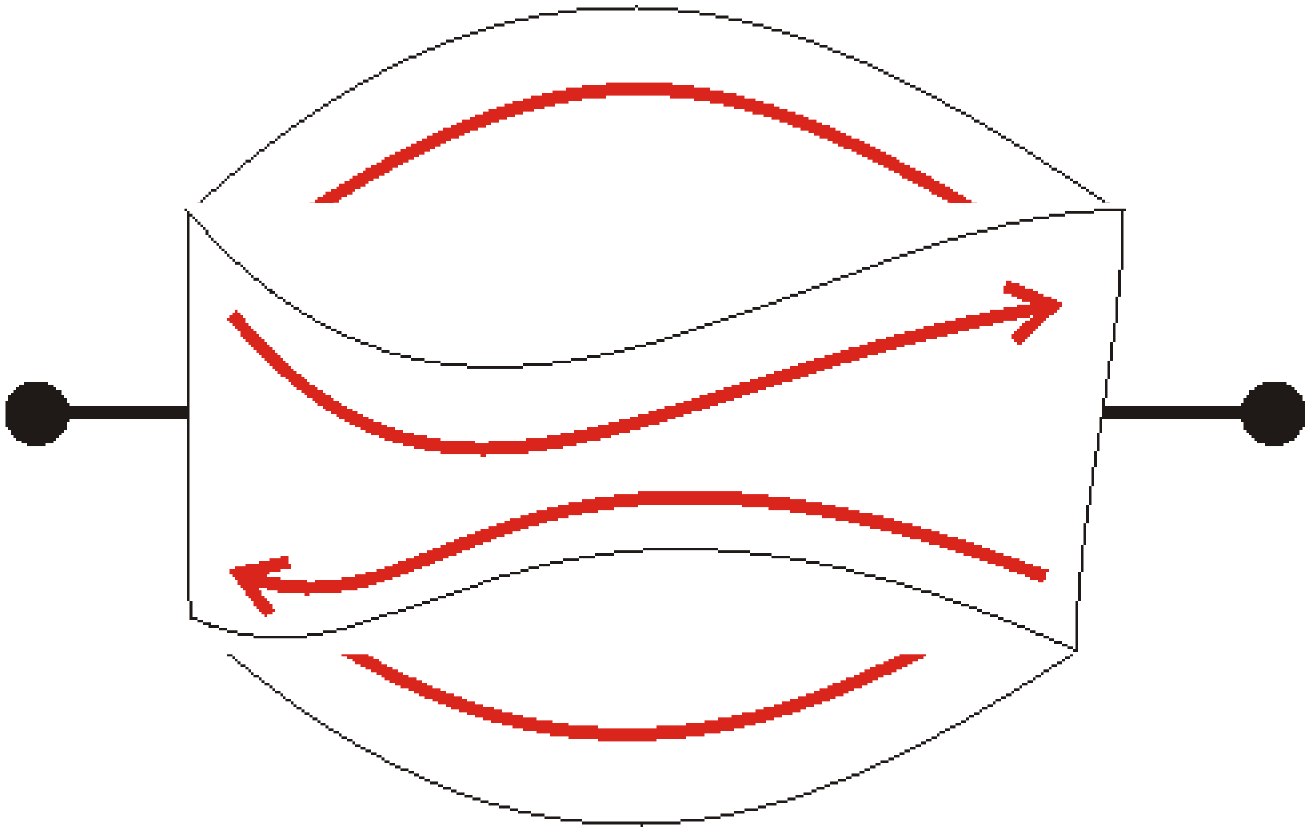


The QHE state spatially separates the **right and left moving states** (spinless 1D liquid)

➔ **suppressed backscattering**

first example of a new Topological Insulator

Topological Insulator



Topological Band Theory

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The distinction between a conventional insulator and the quantum Hall state is a **topological property** of the manifold of occupied states

$H(\mathbf{k})$: Brillouin zone (torus) $\mapsto$ Bloch Hamiltonians with energy gap

Classified by Chern (or TKNN) integer topological invariant (Thouless et al, 1982)

$$n = \frac{1}{2\pi i}\int_{BZ} d^2\mathbf{k} \cdot \langle \nabla_{\mathbf{k}} u(\mathbf{k}) | \times | \nabla_{\mathbf{k}} u(\mathbf{k}) \rangle$$

u(**k**) = Bloch wavefunction

Insulator : n = 0

IQHE state : $\sigma_{xy} = n\, e^2/h$

Geometry and Topology

Gauss and Bonnet

topological

$$\frac{1}{2\pi}\int_S K dA = 2(1-g)$$

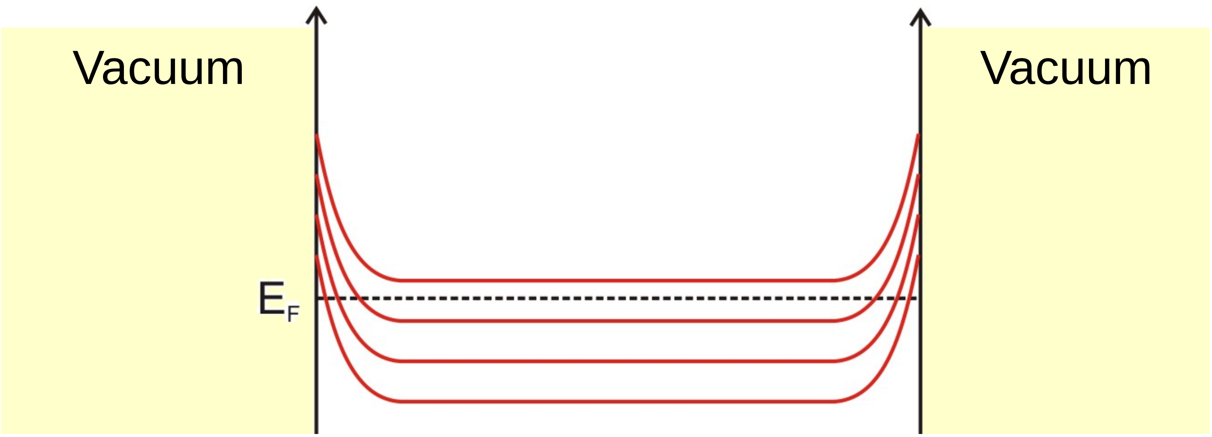
geometric

S: surface
K^{-1}: product of the two radii of curvatures
dA: area element
g: number of handles

Example: **Mug to Torus**

Edge States

The TKNN invariant can only change at a phase transition where the energy gap goes to zero



n = 0 n = 3 n = 0

Δn = # Chiral Edge Modes

This approach can actually be generalized to a spinfull QHE at zero magnetic field: the Quantum Spin Hall Effect

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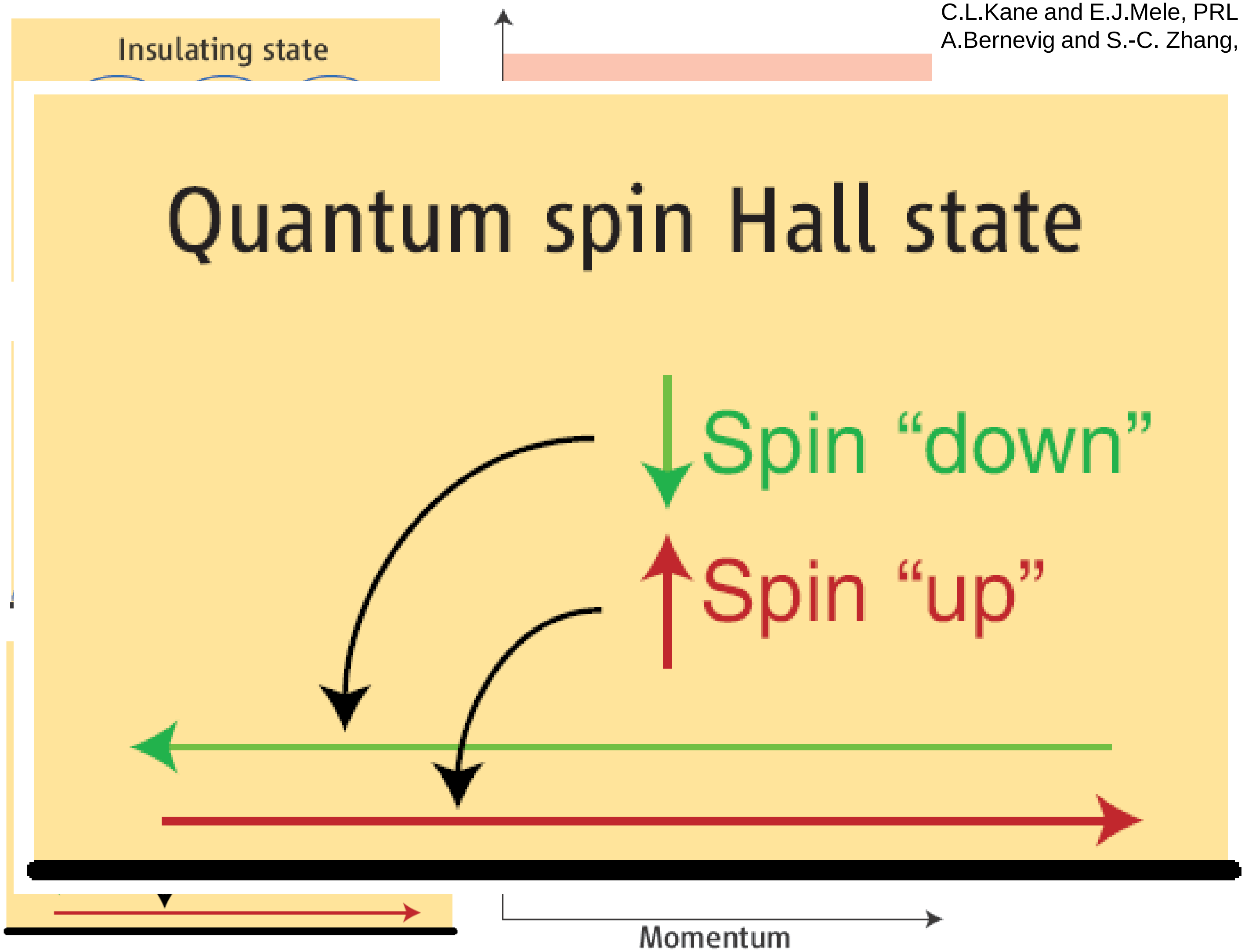
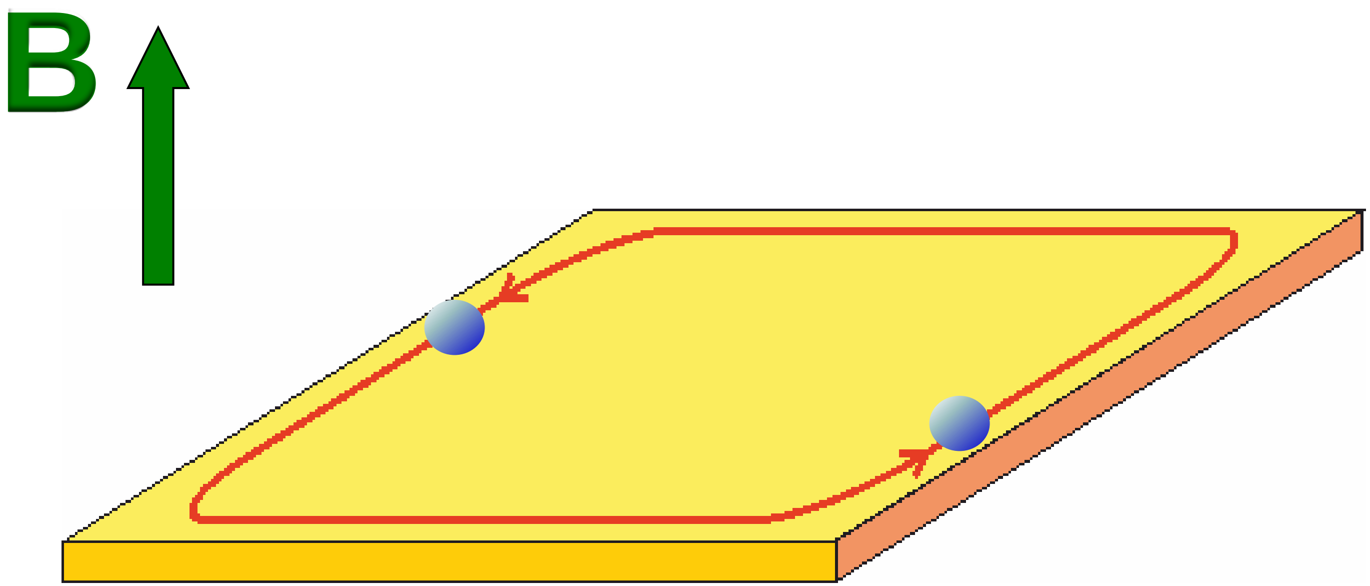


C.L.Kane and E.J. Mele, Science **314,** 1692 (2006)

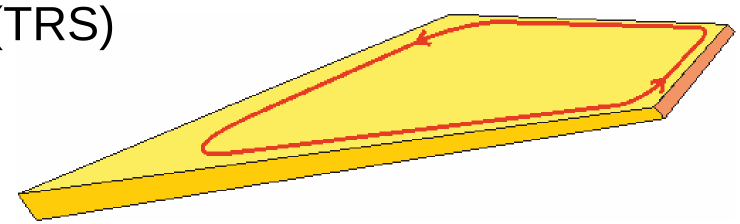
A two-dimensional topological insulator



quantum Hall effect

topological invariant

broken time reversal symmetry (TRS)



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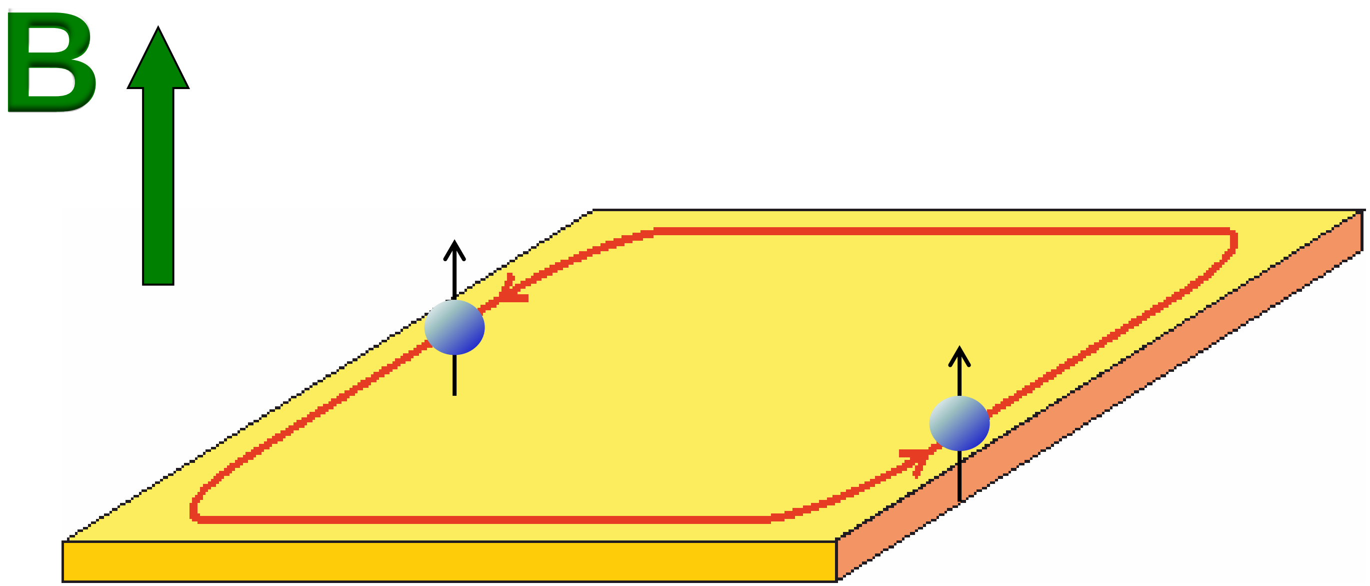

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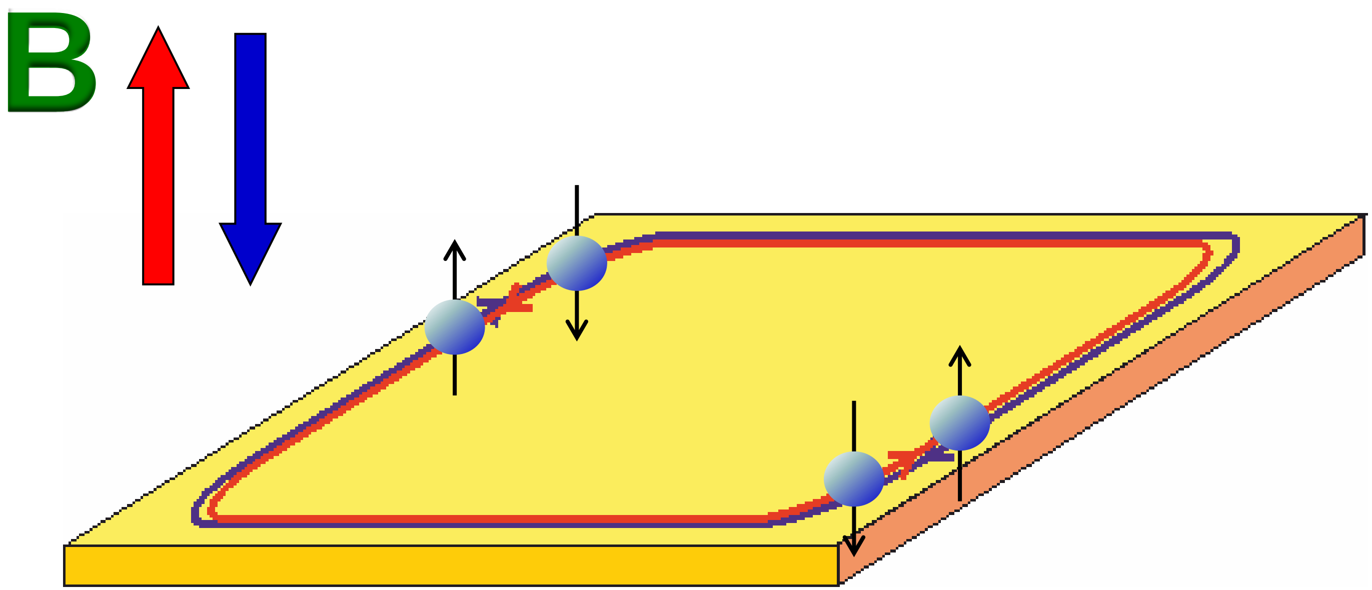

helical edge states

invariant under time reversal

Protected Edge States

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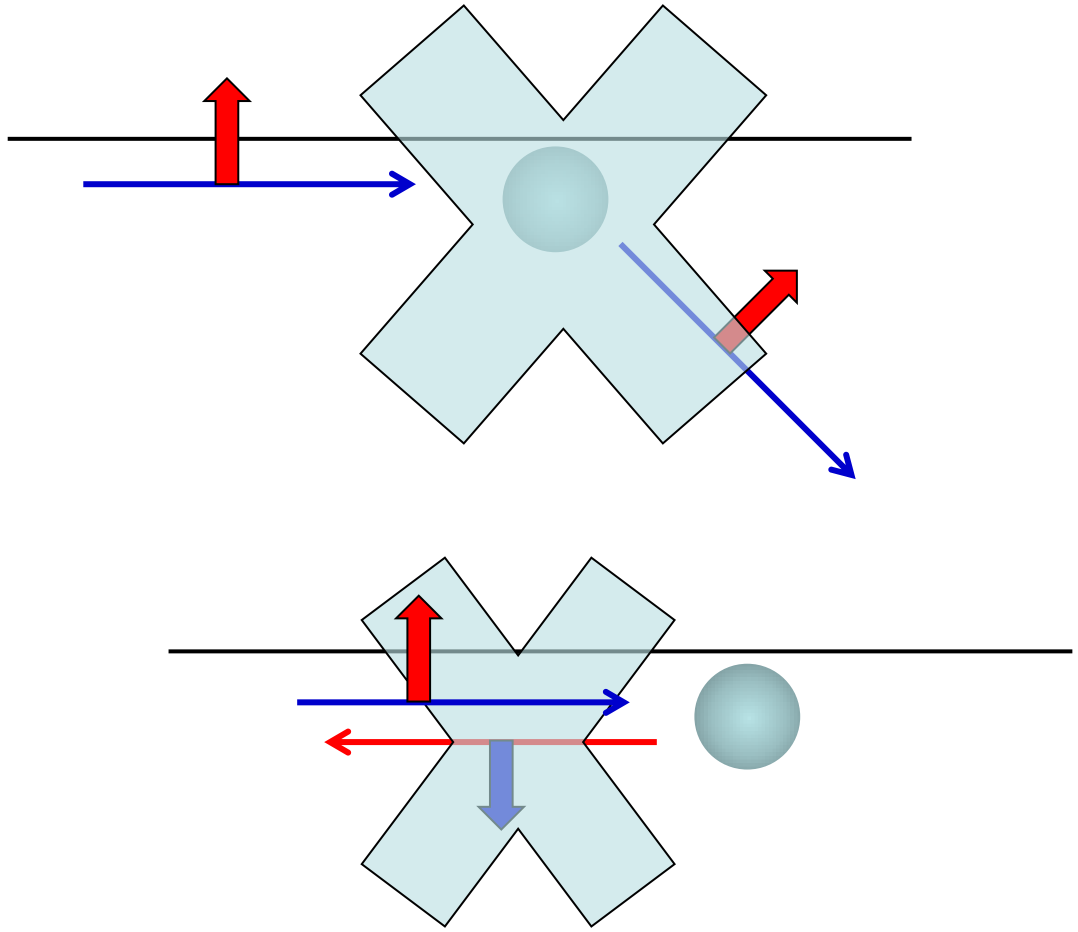
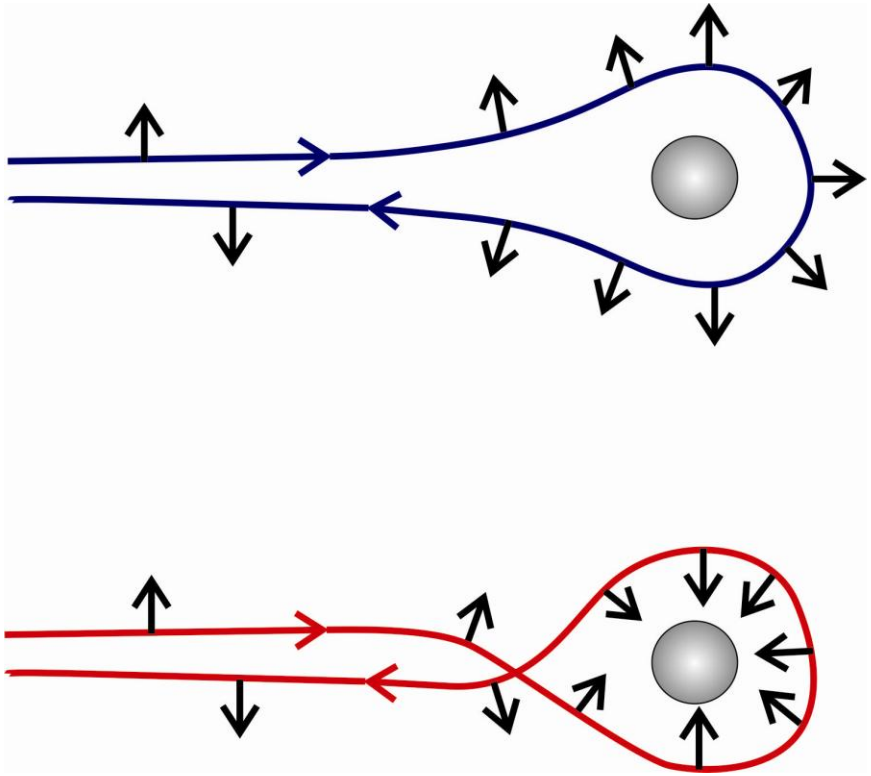
suppressed backscattering:



Protected Edge States

suppressed backscattering:



spin rotation:

π

$-\pi$

$\Delta\varphi = 2\pi$

➔ Interference is destructive

Quantum Spin Hall Effect

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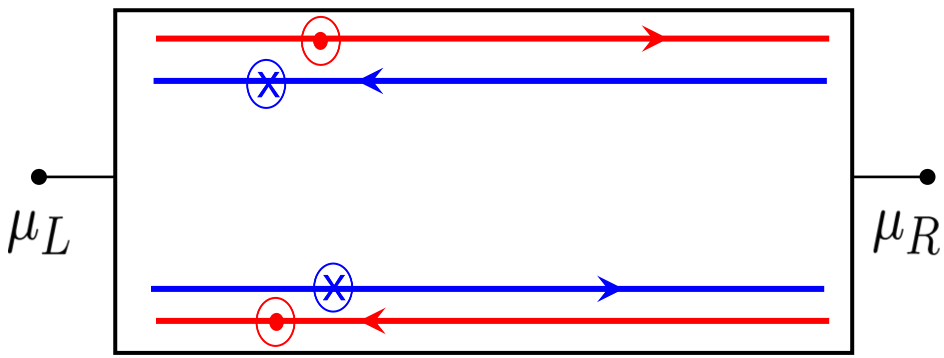
Stability of Helical Edge States:

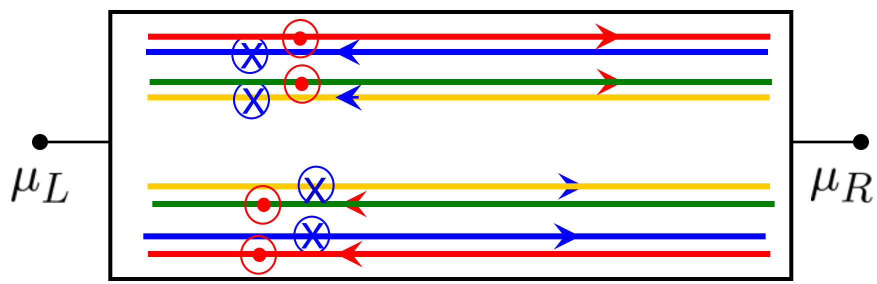


$$G_{QSHE} = \frac{2e^2}{h}$$

backscattering is only possible by **time reversal symmetry breaking processes**
(for example external magnetic fields)

or if **more states** are present

$\psi_{k_1,s+}, \psi_{-k_2,s-}$



Topological Insulator : A New B=0 Phase

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$$H = \begin{pmatrix} h(\vec{k}) & 0 \\ 0 & h^*(-\vec{k}) \end{pmatrix} \quad \text{with} \quad h(\vec{k}) = \varepsilon(\vec{k}) + d_a(\vec{k})\sigma^a$$

TRS is preserved!

TKNN invariant for **one** Kramers partner: $$n_\uparrow = \frac{1}{4\pi}\int d\mathbf{k}\left(\partial_{k_x}\hat{d} \times \partial_{k_y}\hat{d}\right)\cdot\hat{d}$$

zero Hall conductivity:

$$n = n_\uparrow + n_\downarrow$$

quantized spin Hall conductivity:

$$n_\sigma = \frac{n_\uparrow - n_\downarrow}{2}$$

$\Rightarrow$ **Z_2 invariant:** $$\nu = n_\sigma \bmod 2$$

# of Kramers pairs at edge:

$$N_K = \Delta\nu \bmod 2$$

Avron et al PRL 1988; *Kane & Mele* PRL 2005; *Sheng et al* PRL 2006; *Hasan & Kane* RMP 2010

Topological Insulator : A New B=0 Phase

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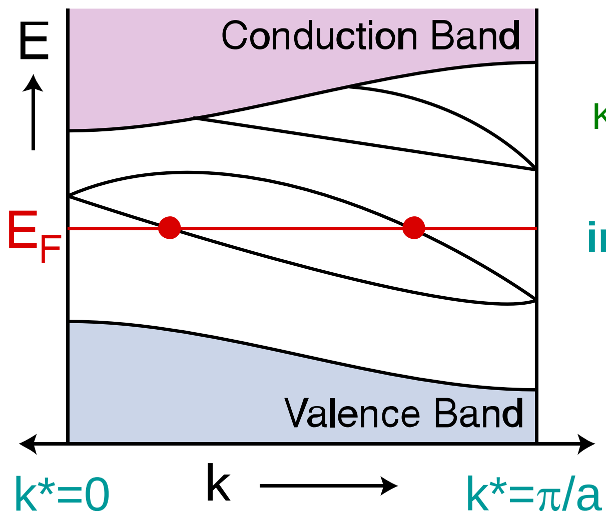
There are 2 classes of 2D time reversal invariant band structures

Z_2 topological invariant: $\nu = 0,1$

Edge States for $0<k<\pi/a$

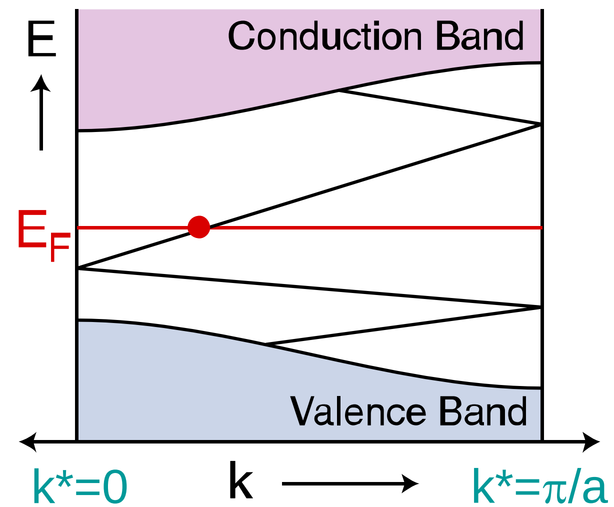
$\nu=0$: Conventional Insulator



even number of bands
crossing Fermi energy

Kramers degenerate at
time reversal
invariant momenta
$k^* = -k^* + G$

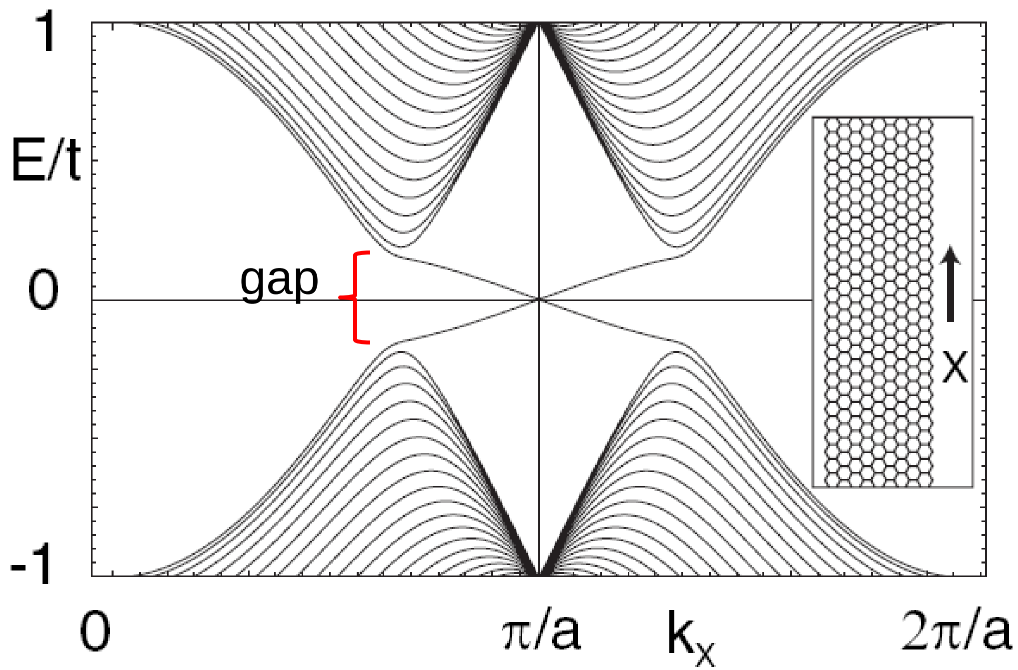
$\nu=1$: Topological Insulator



odd number of bands
crossing Fermi energy

QSHE in Graphene

Graphene edge states

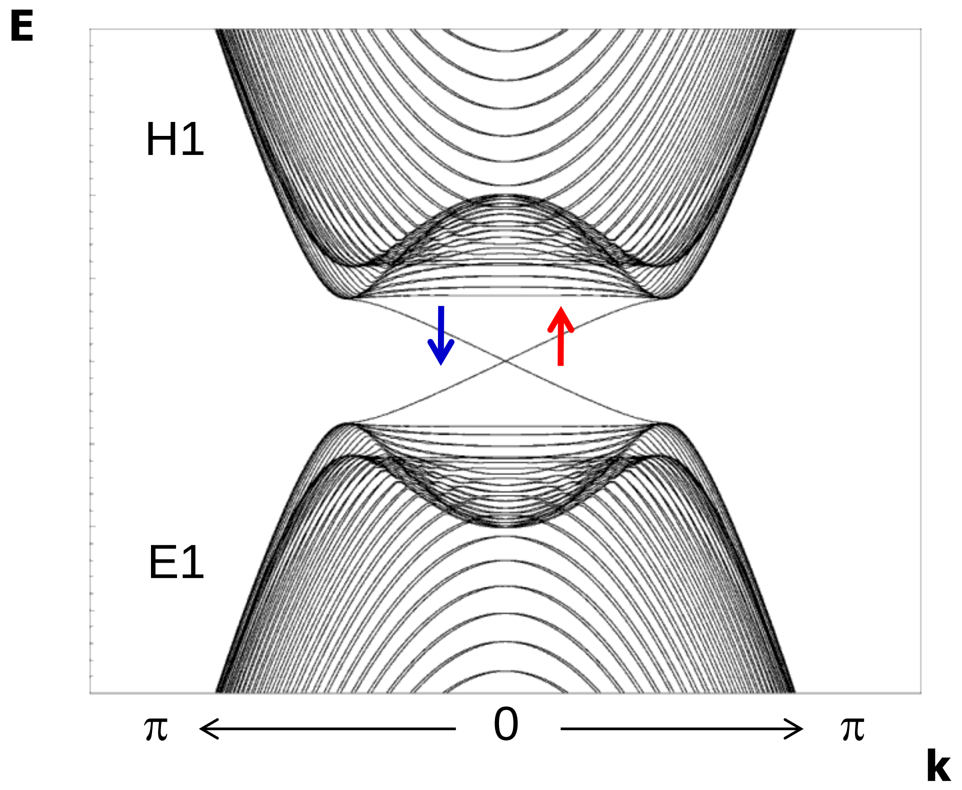


C.L.Kane and E.J.Mele, PRL **95**, 226801 (2005)

- Graphene – spin-orbit coupling strength is too weak ➔ gap only about 30 μeV.
- ➔ **not** accessible in experiments

QSHE in HgTe

Helical edge states for inverted HgTe QW



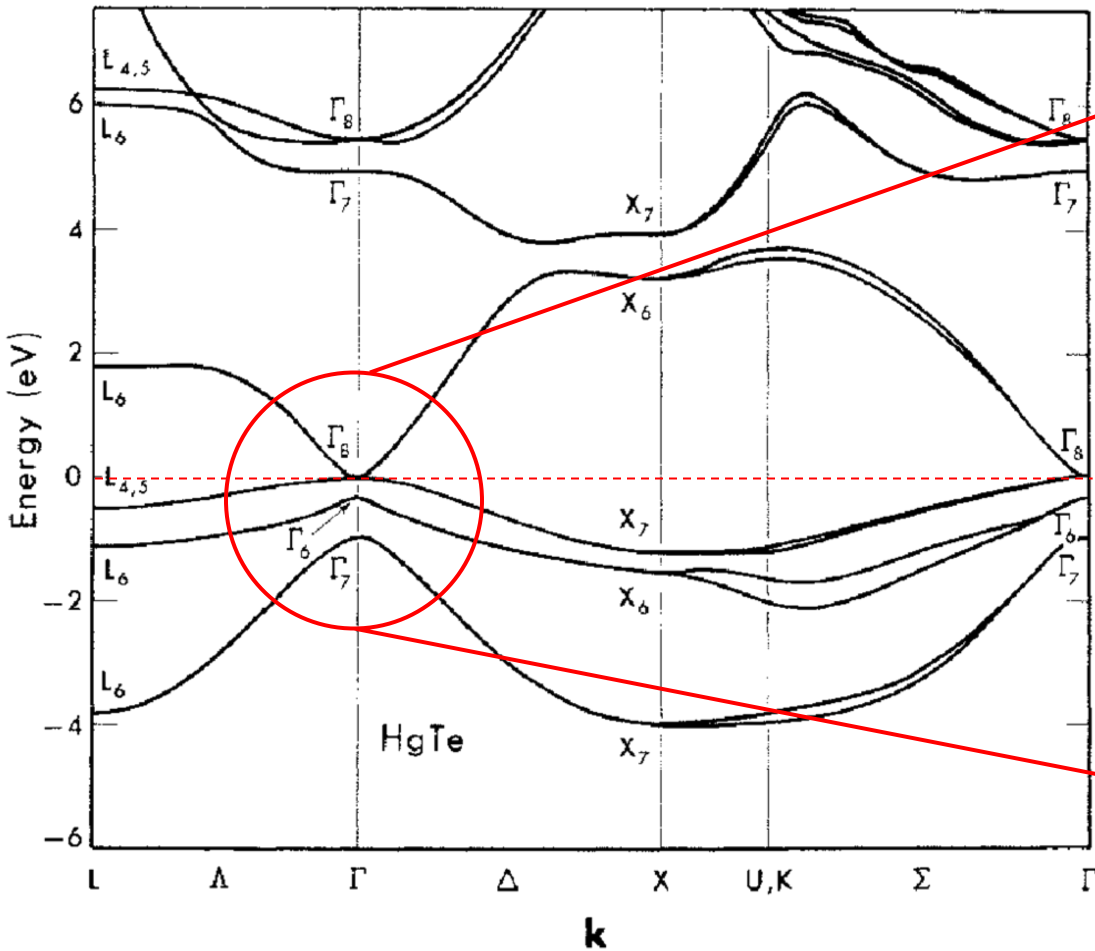
B.A Bernevig, T.L. Hughes, S.C. Zhang, Science **314**, 1757 (2006)

HgTe-Quantum Well Structures

HgTe

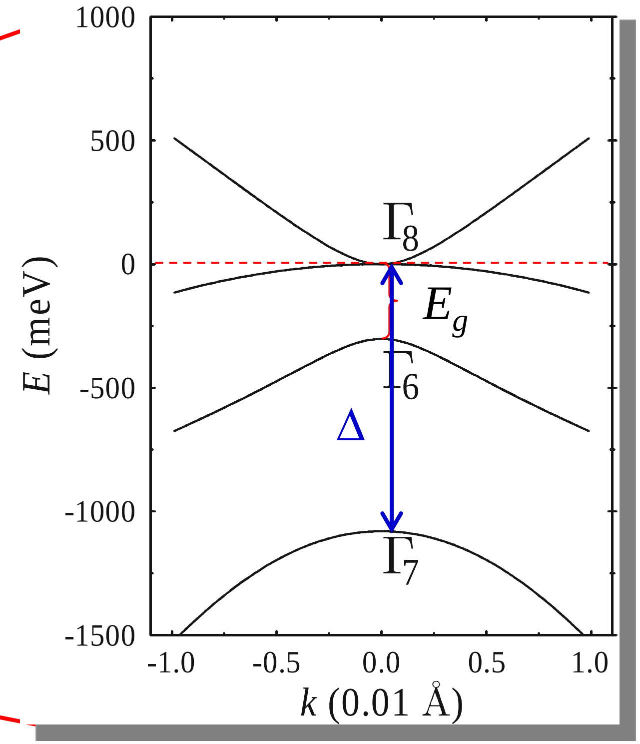
EP 3
H. Buhmann

band structure



D.J. Chadi et al. PRB, 3058 (1972)

semi-metal



semiconductor

fundamental energy gap

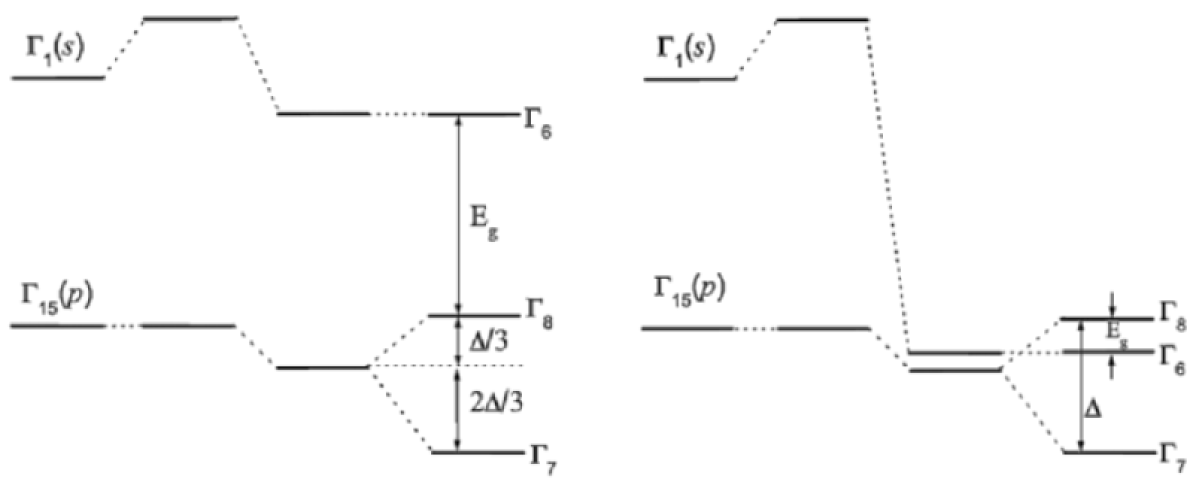
$$E^{\Gamma 6} - E^{\Gamma 8} \approx -300\,\text{meV}$$

CdTe and HgTe

Fine structure (first order relativistic corrections):

- Darwin-Term: Electron Compton Wavelength $\lambda_c = \frac{\hbar}{mc}$
- Mass-Velocity-Correction: $M_{Hg} = 200.6$ u $M_{Cd} = 112.4$ u
- Spin-Orbit-Coupling: Dominated by Telluride
- $E_{Gap} = \Gamma_6 - \Gamma_8$

CdTe HgTe

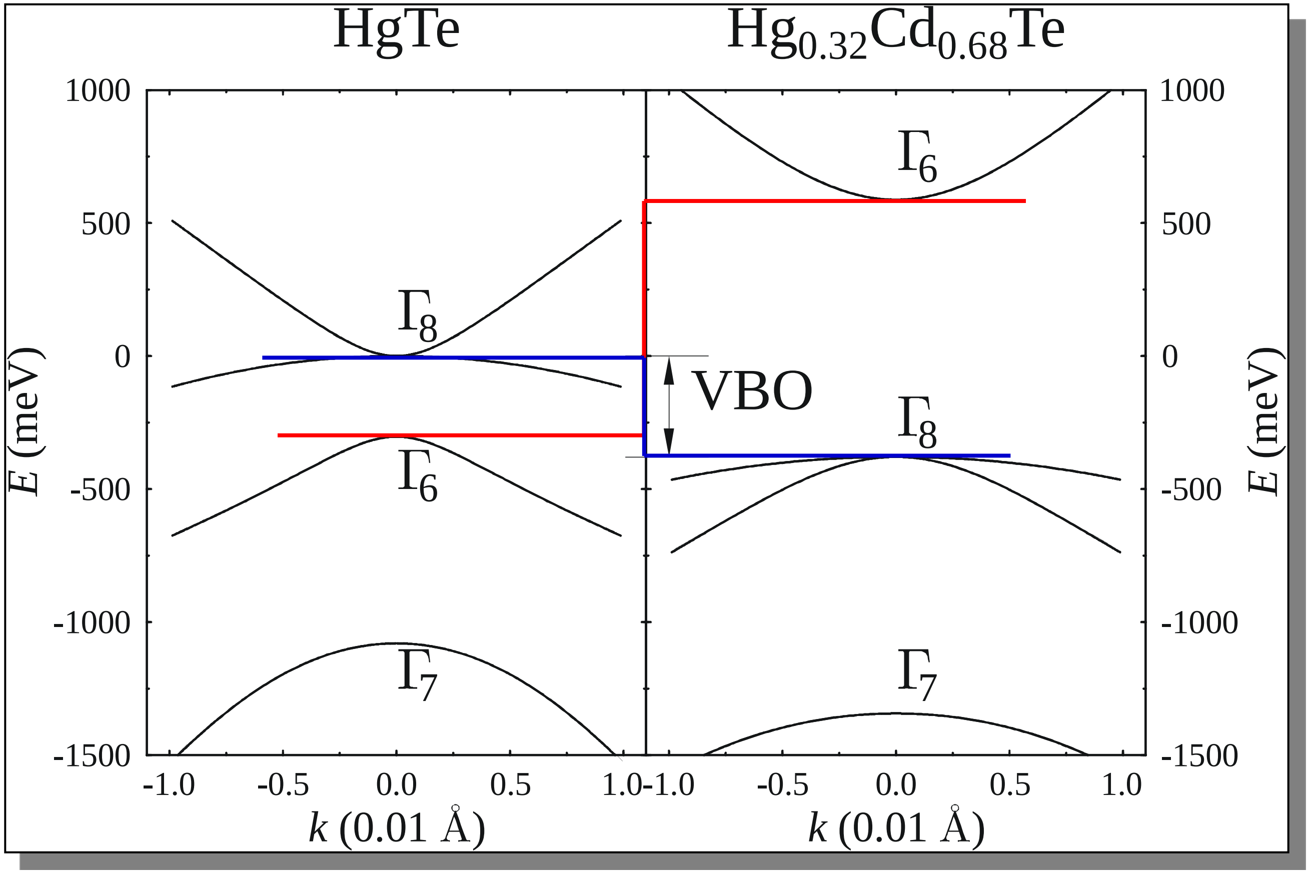


$$H = H_1 + H_D + H_{mv} + H_{so}$$

HgTe-Quantum Wells

QW

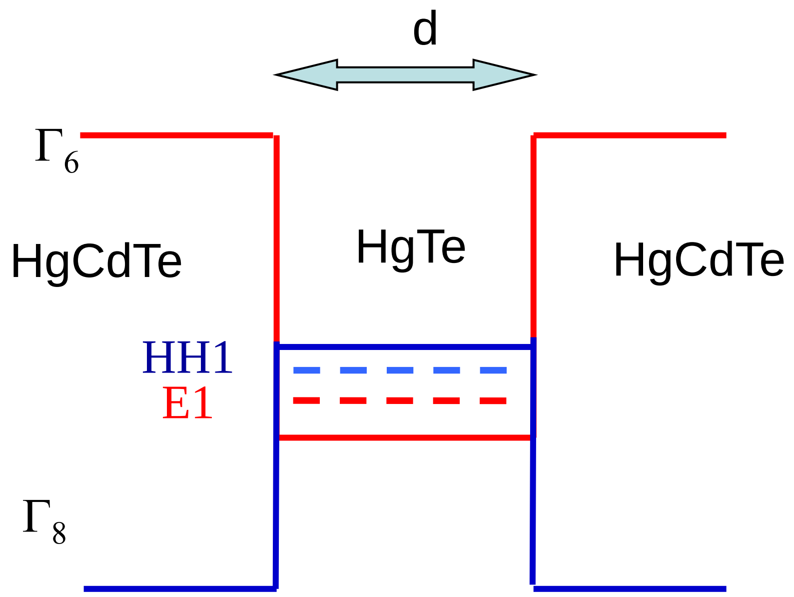
Barrier



VBO = 570 meV

HgTe-Quantum Wells

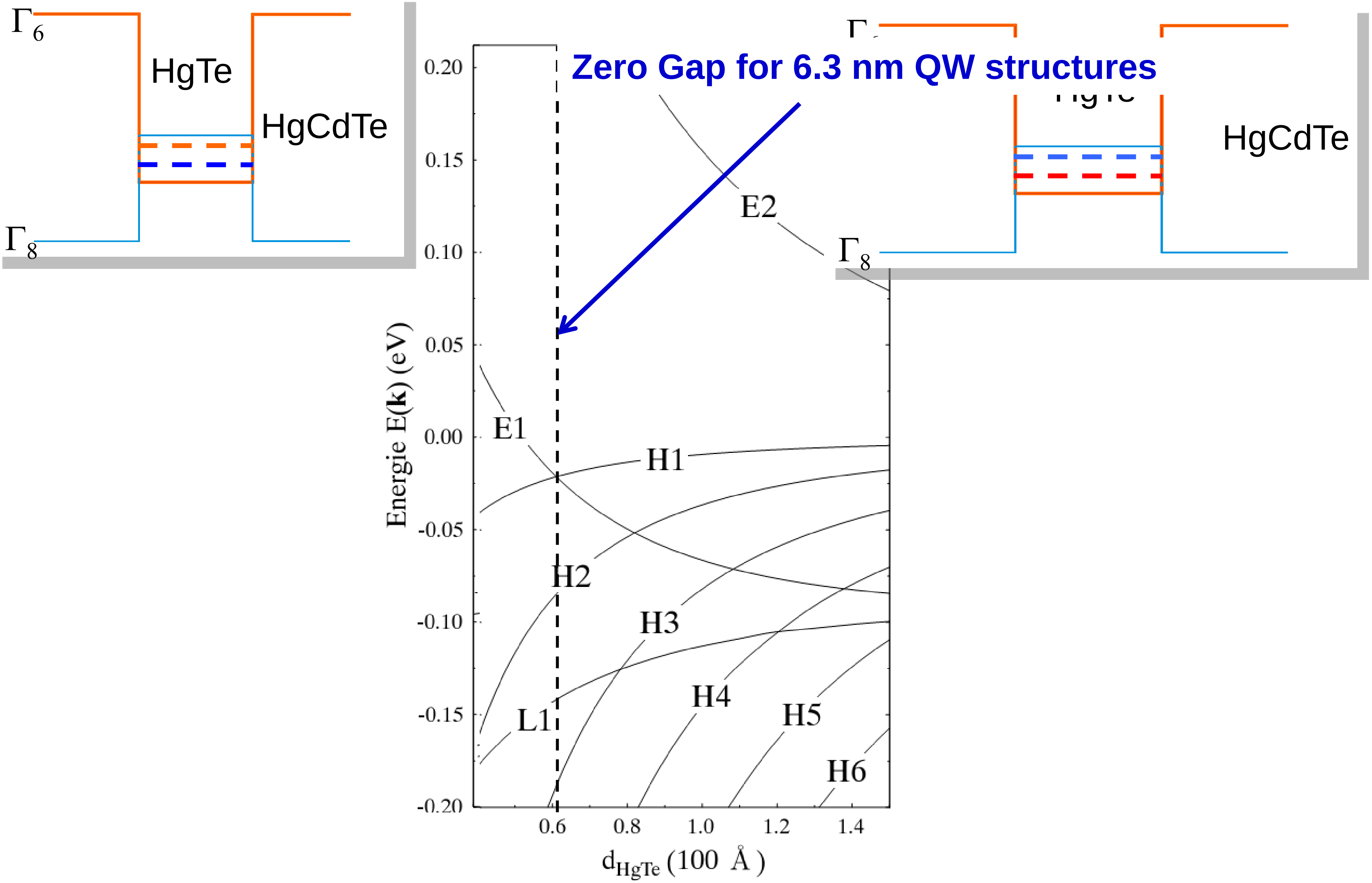
Typ-III QW



normal
inverted

band structure

Bandstructure



Dirac Band Structure

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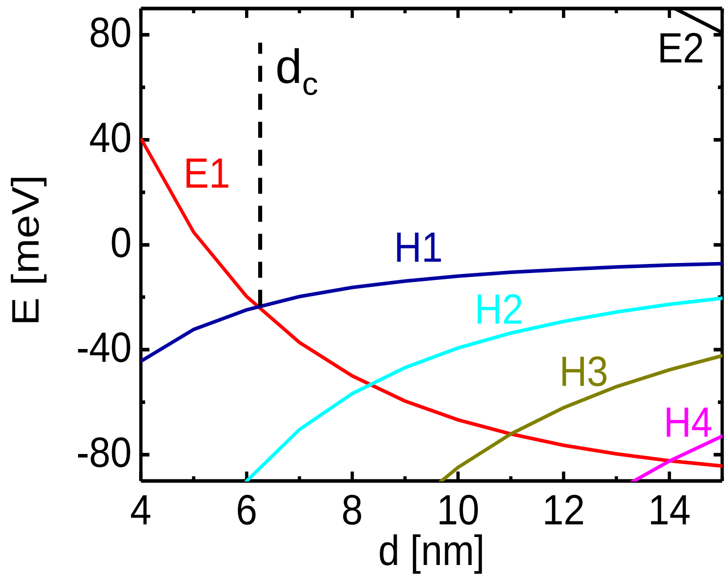
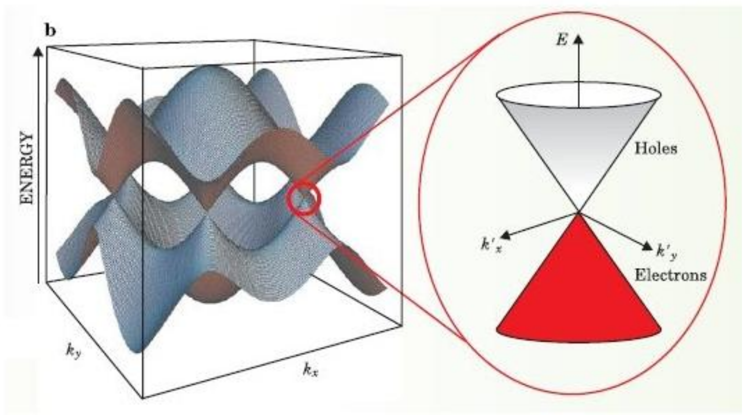
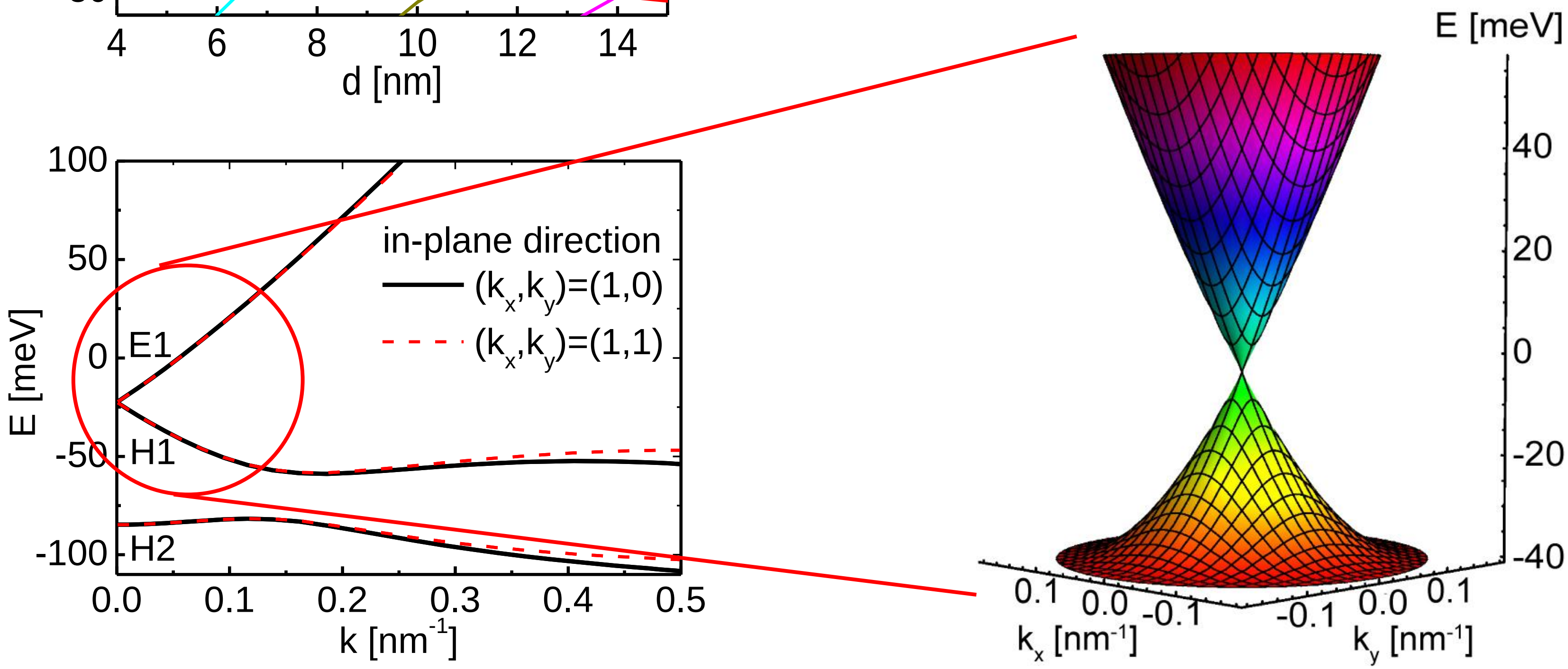
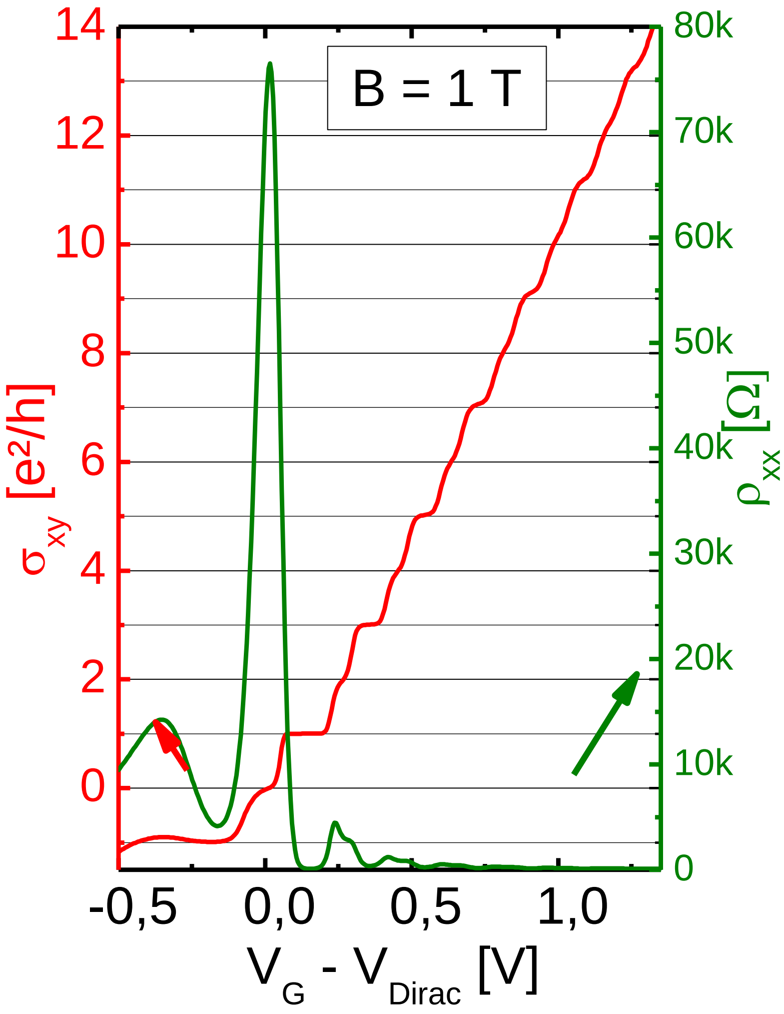
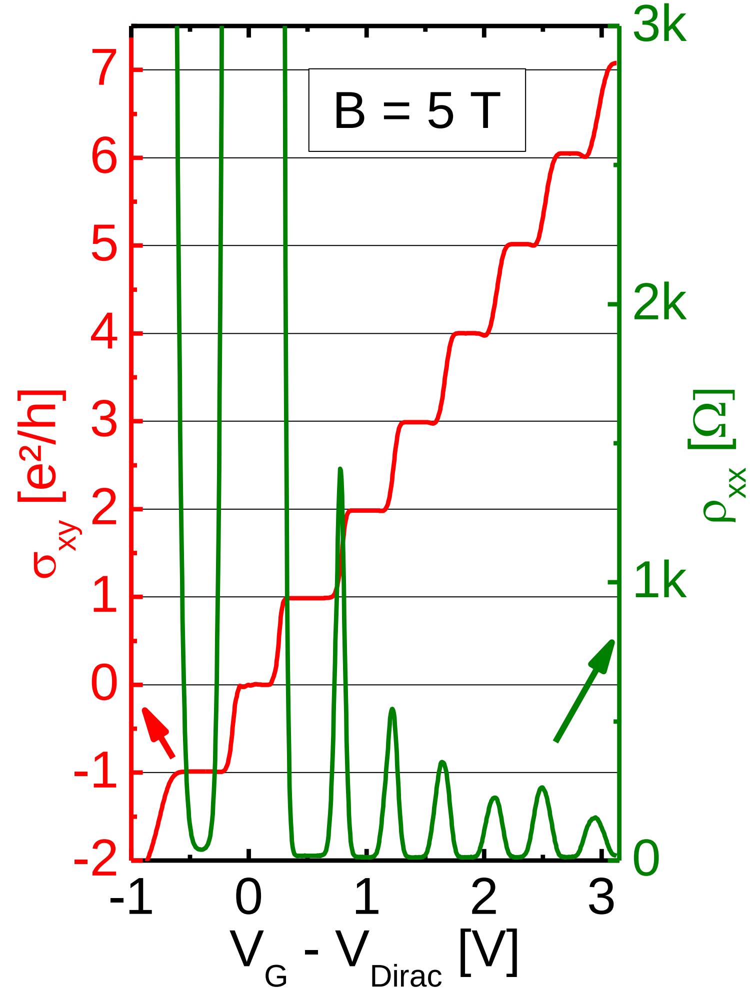






Zero Gap





B. Büttner et al., Nature Physics **7**, 418 (2011)

LL-Dispersion

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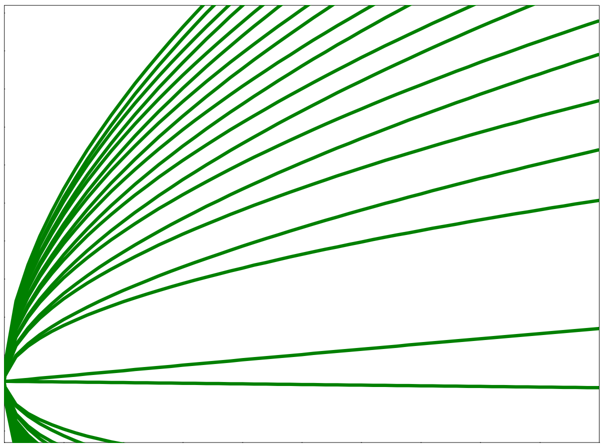
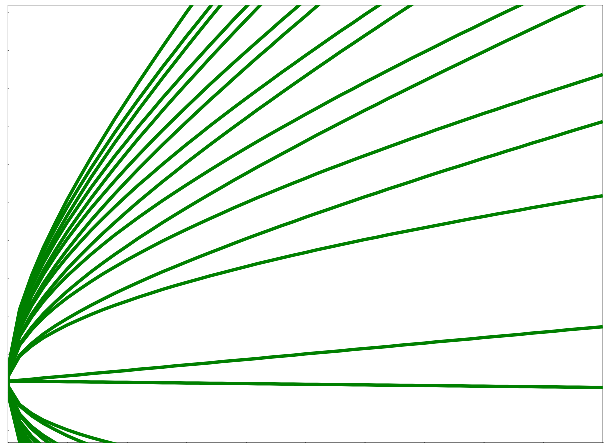
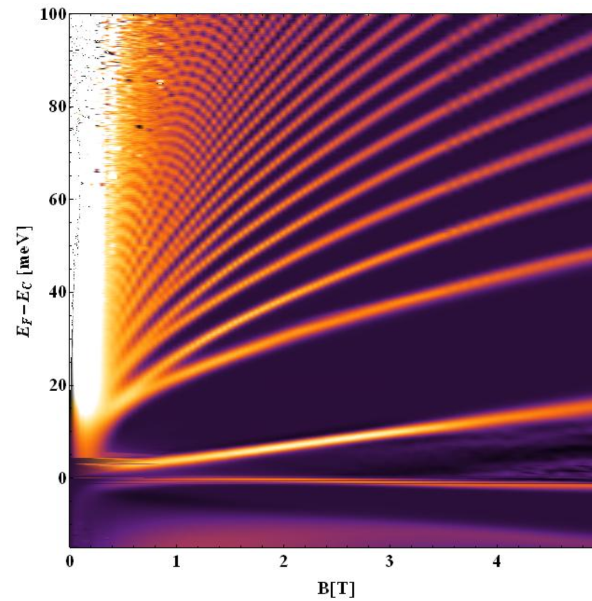
Kane model



effective model



LLchart

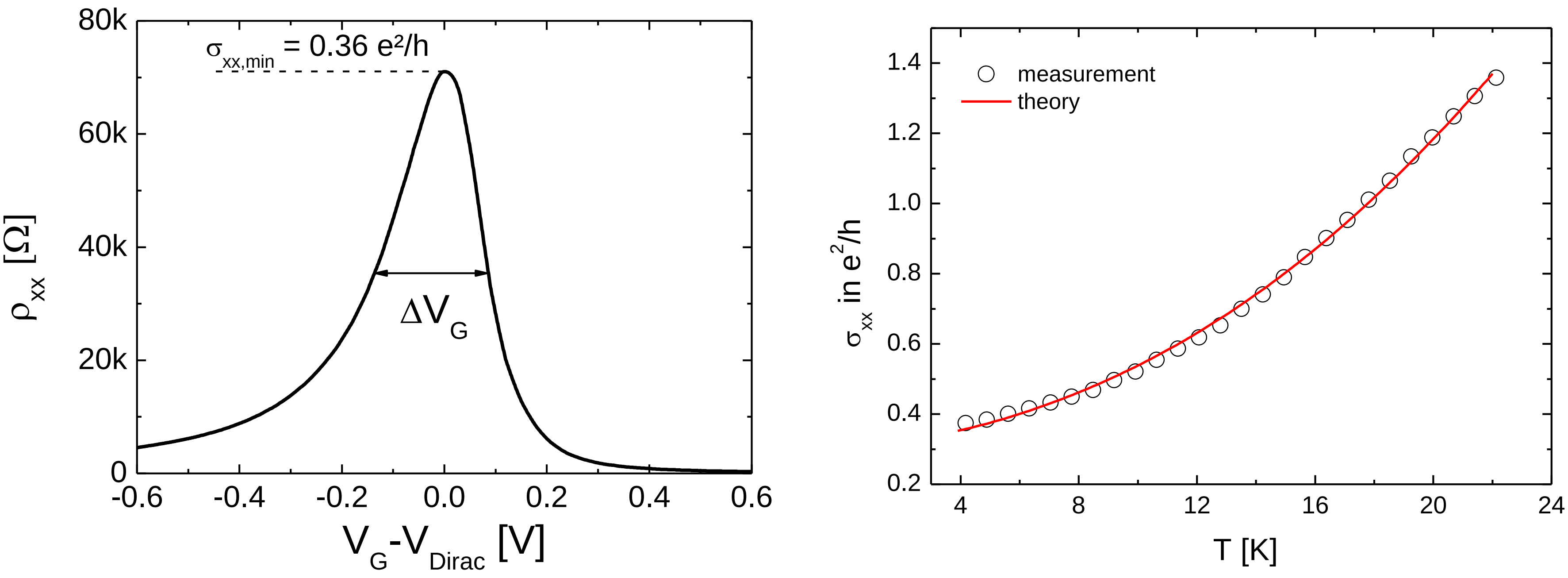


Parameters:
M ~ 0 (-0.035 meV)
D = -682.2 meV nm²
G = -857.3 meV nm²
A = 372.9 meV nm

E(VG) conversion taken from appropriate Kane model

B. Büttner et al., Nature Physics **7**, 418 (2011)

Finite Mobility



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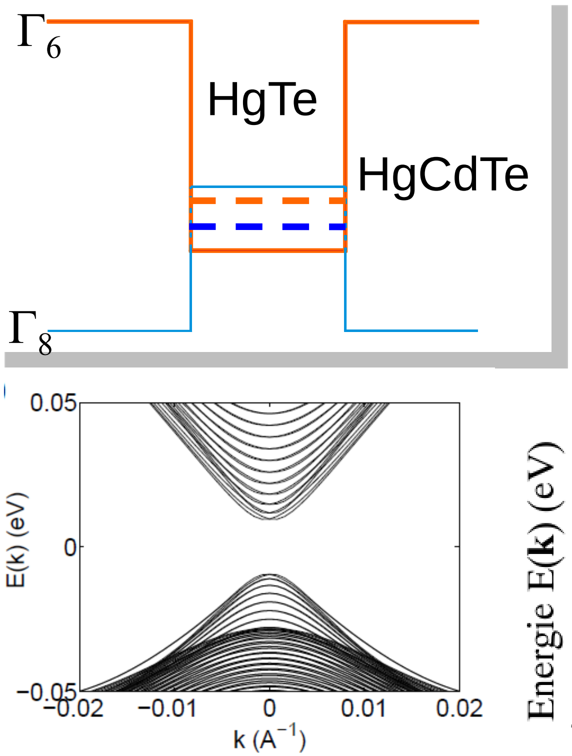
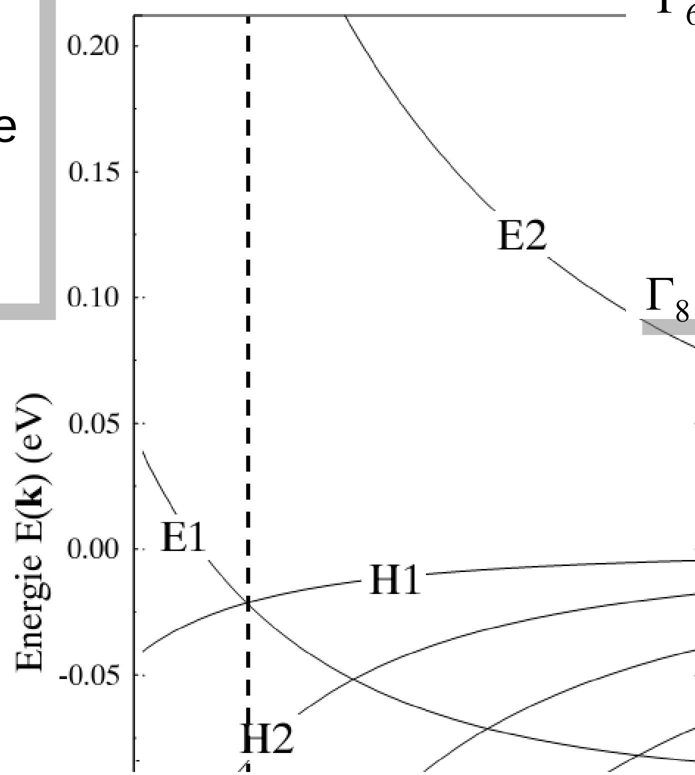
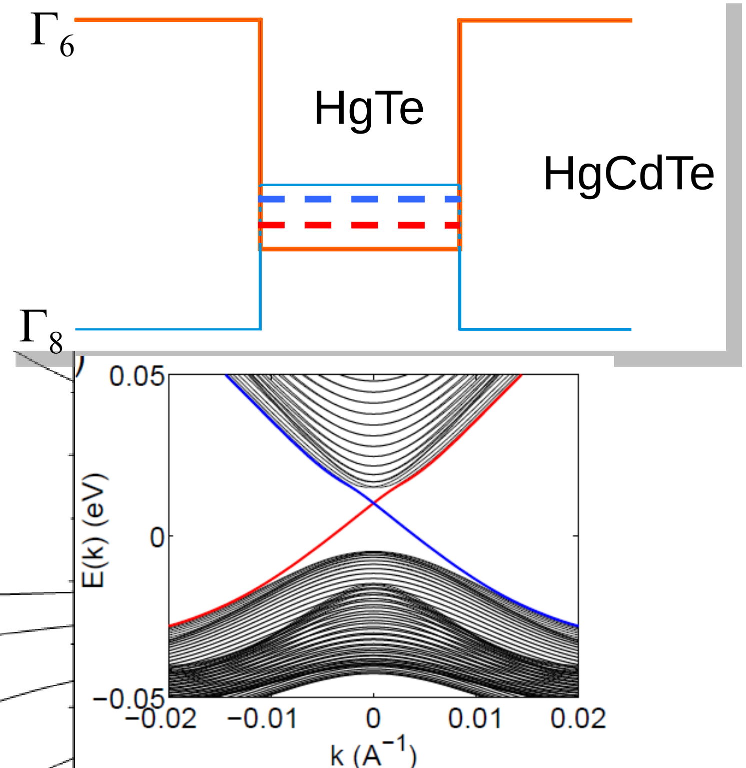
Bandstructure







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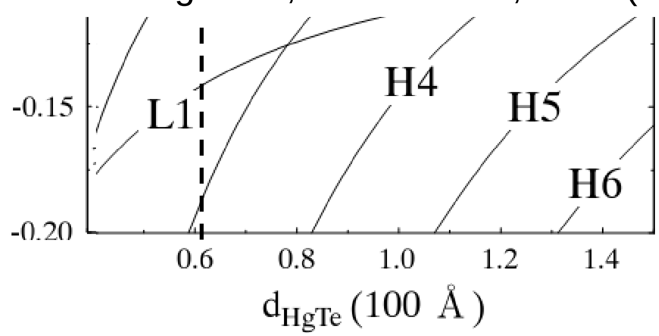
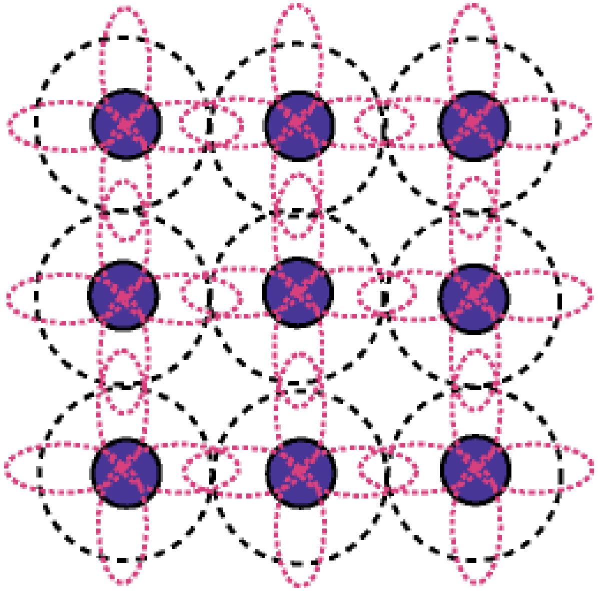


Effective Tight Binding Model

square lattice with 4 orbitals per site

$$|s,\uparrow\rangle, |s,\downarrow\rangle, |(p_x+ip_y),\uparrow\rangle, |-(p_x-ip_y),\downarrow\rangle$$



nearest neighbor hopping integral

basis: $|E_1+\rangle, |H_1+\rangle, |E_1-\rangle, |H_1-\rangle$

$$H_{eff}(k_x,k_y) = \begin{pmatrix} h(k) & 0 \\ 0 & h^*(-k) \end{pmatrix}$$

Effective Tight Binding Model

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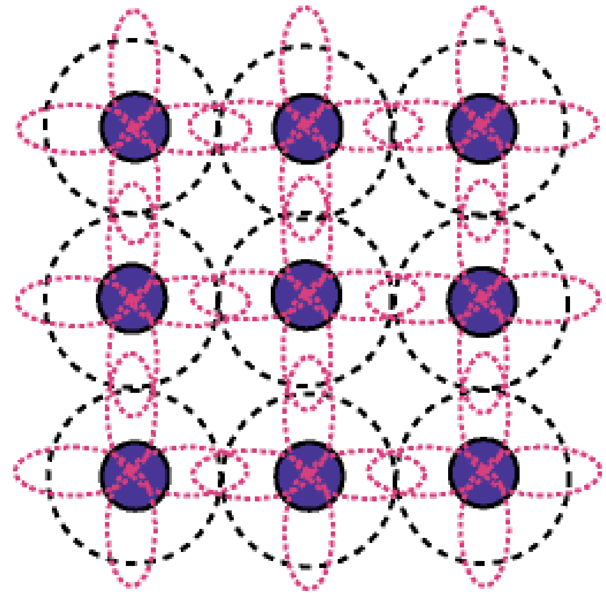
square lattice with 4 orbitals per site

$$|s,\uparrow\rangle, |s,\downarrow\rangle, |(p_x + ip_y),\uparrow\rangle, |-(p_x - ip_y),\downarrow\rangle$$



nearest neighbor hopping integral

basis: $|E_1 +\rangle, |H_1 +\rangle, |E_1 -\rangle, |H_1 -\rangle$

$$H_{eff}(k_x, k_y) = \begin{pmatrix} h(k) & 0 \\ 0 & h^*(-k) \end{pmatrix}$$

$$h(k) = \begin{pmatrix} M(k) + \beta k^2 & A(\sin k_x - i\sin k_x) \\ A(\sin k_x + i\sin k_x) & -M(k) - \beta k^2 \end{pmatrix}$$

at the Γ-point

$$k \to 0 \Rightarrow \begin{pmatrix} M(k) & A(k_x - ik_x) \\ A(k_x + ik_x) & -M(k) \end{pmatrix}$$

relativistic Dirac equation

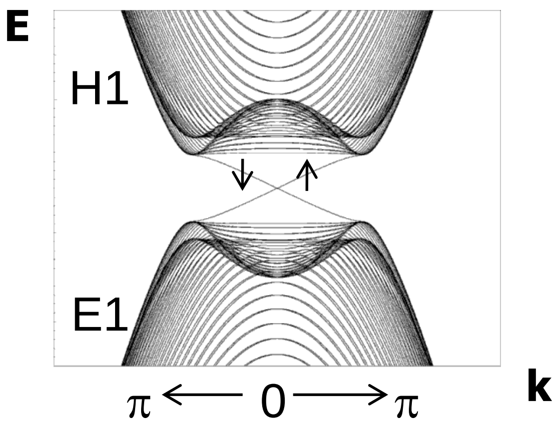
HgTe

$$\left.\begin{matrix} M > 0 \\ M < 0 \end{matrix}\right\} \text{for} \left\{\begin{matrix} \text{normal QW} \\ \text{inverted QW} \end{matrix}\right.$$

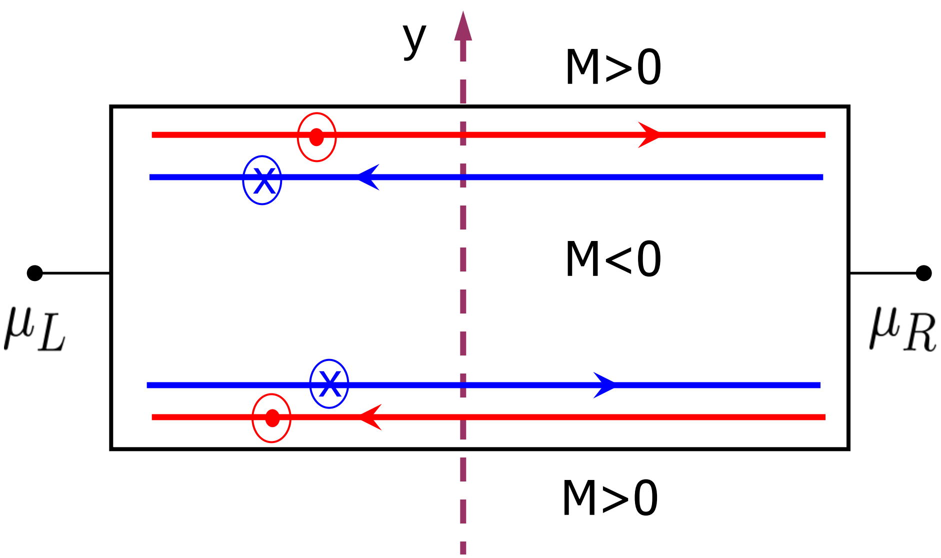
with tunable parameter M (band-gap)

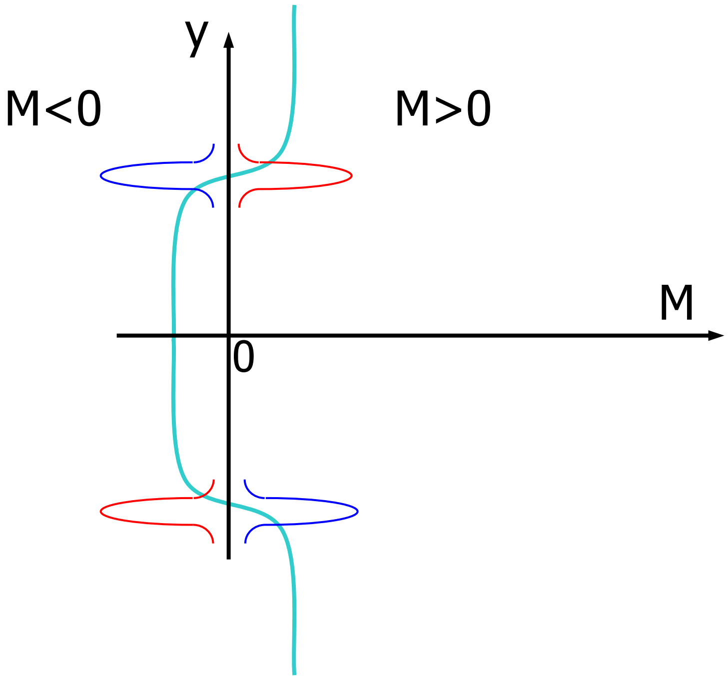
Mass domain wall

inverted
band structure



states localized on the domain wall
which disperse along the x-direction



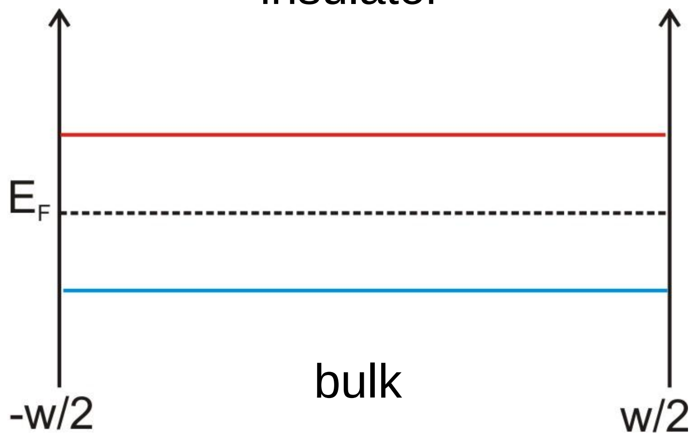


helical edge states

Quantum Spin Hall Effect

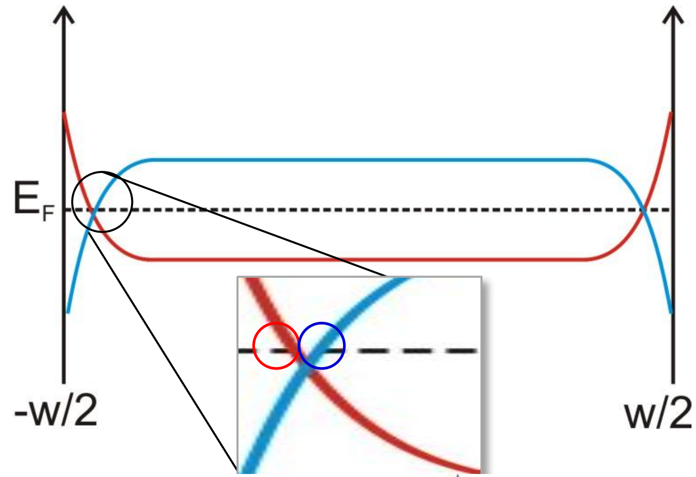
M > 0

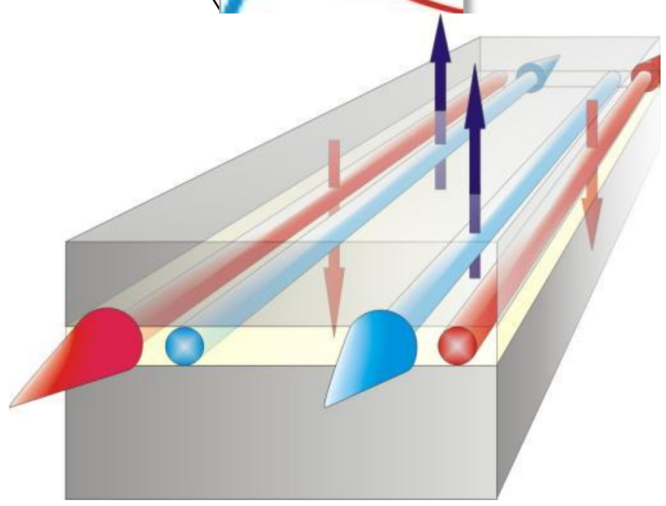
normal insulator



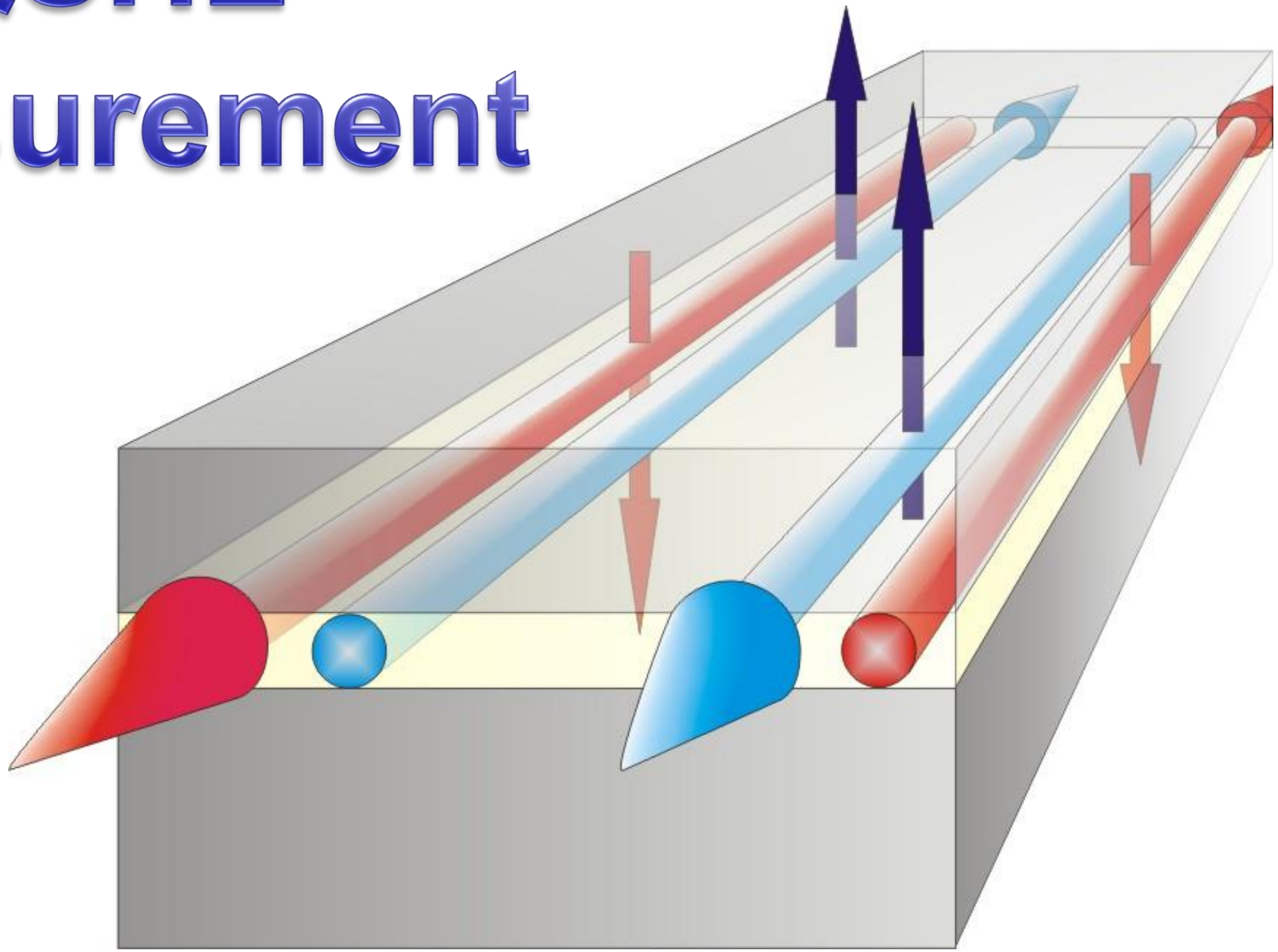
M < 0

QSHE

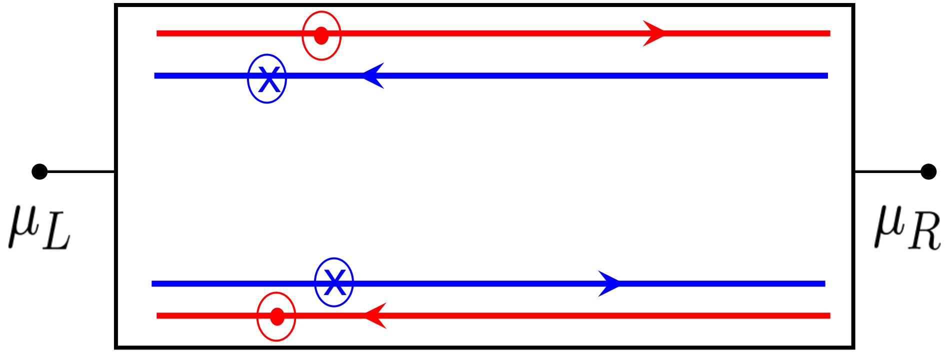




QSHE Measurement



QSHE Measurement

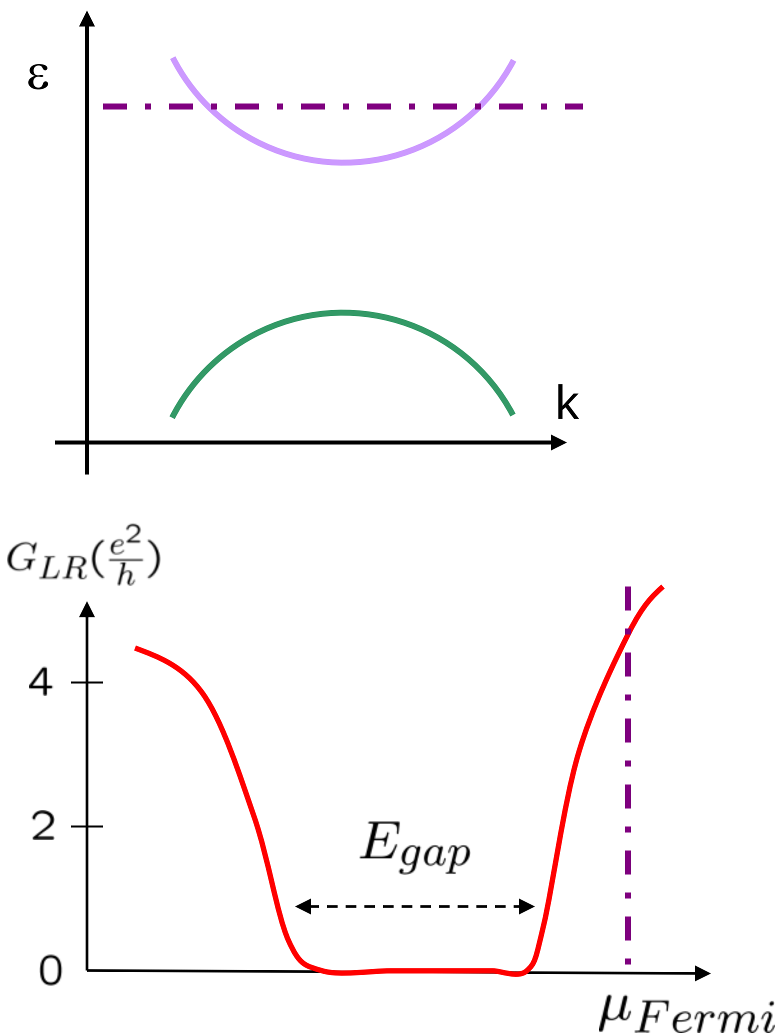


$$G = \frac{2e^2}{h}$$

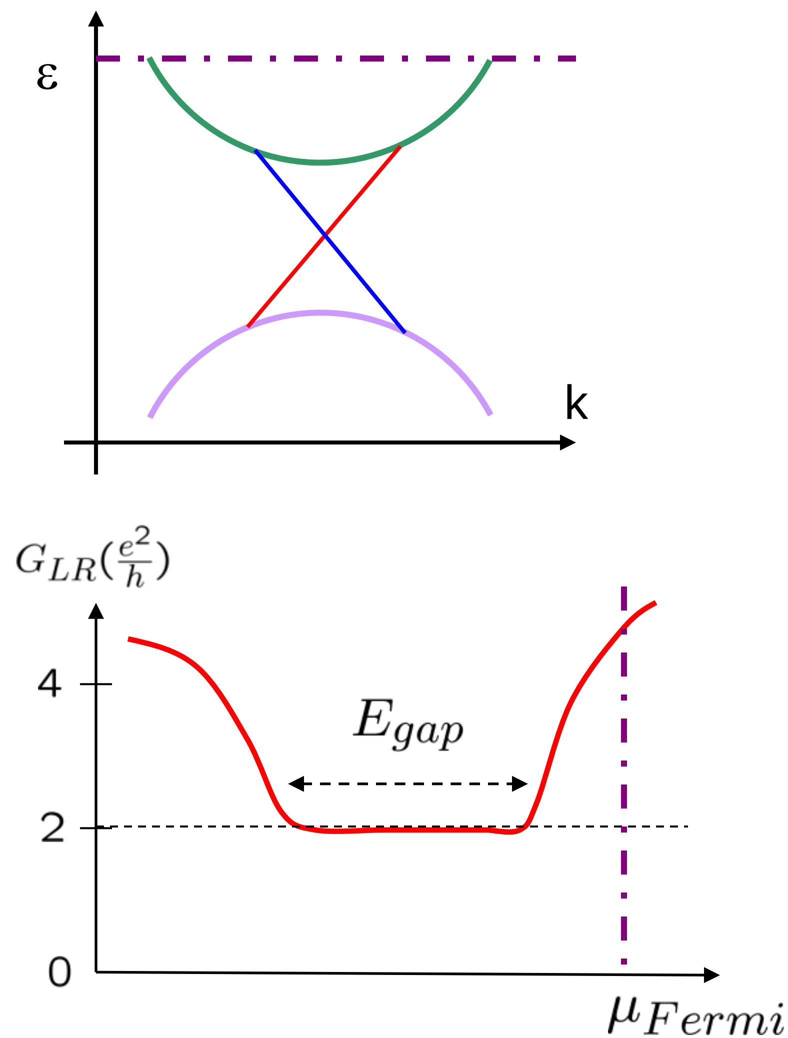
Experiment

EP 3
H. Buhmann

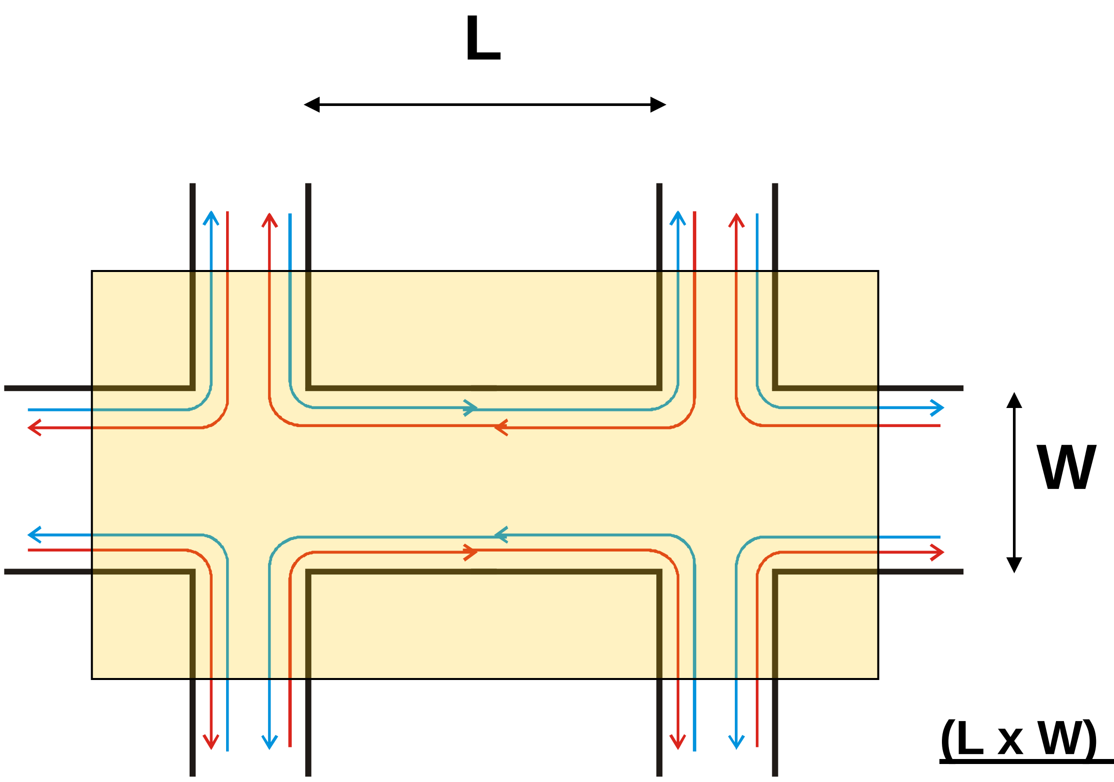
$d < d_c$, normal regime



$d > d_c$, inverted regime



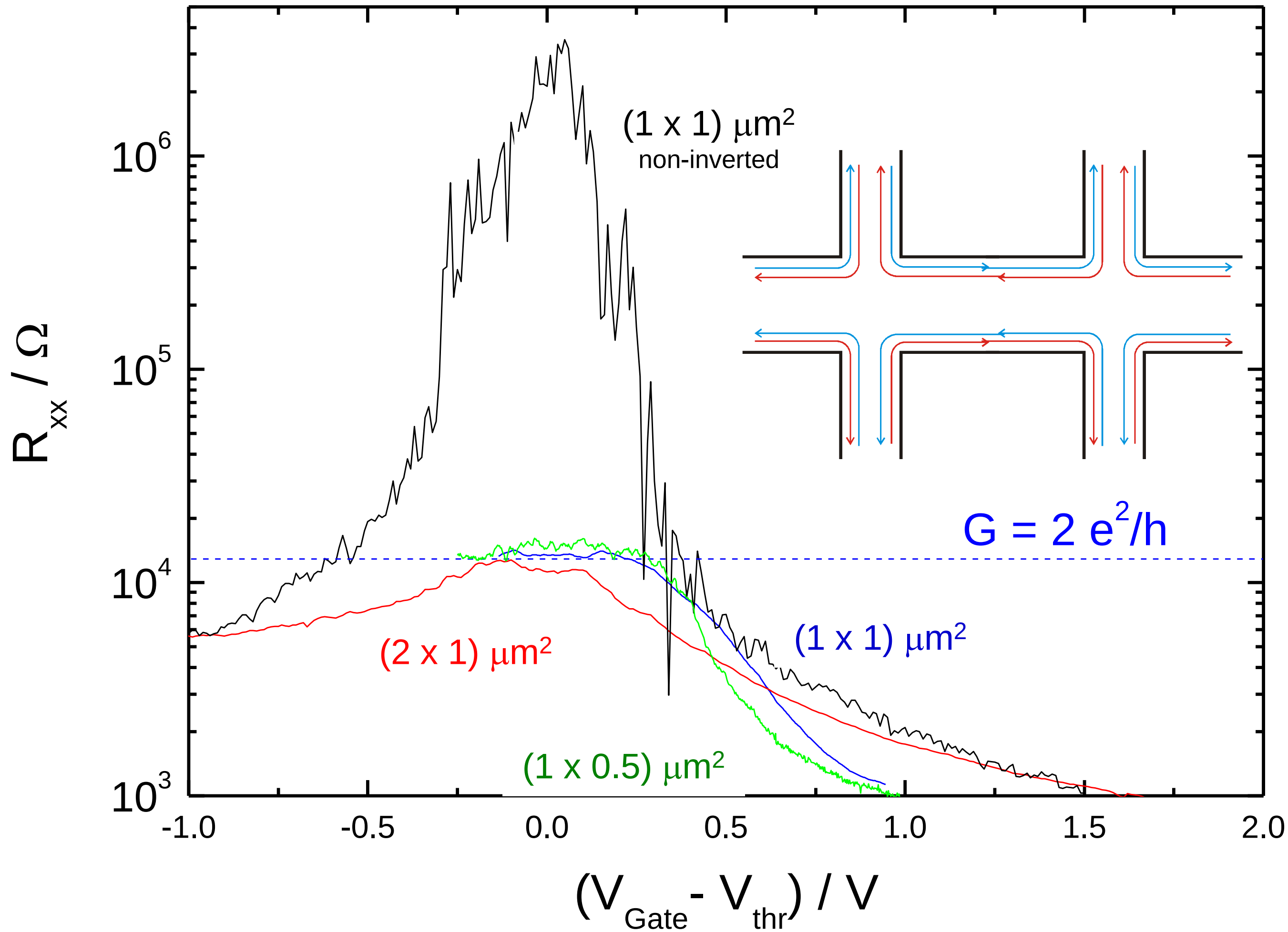
Small Samples



(L x W) μm

2.0 x 1.0 μm
1.0 x 1.0 μm
1.0 x 0.5 μm

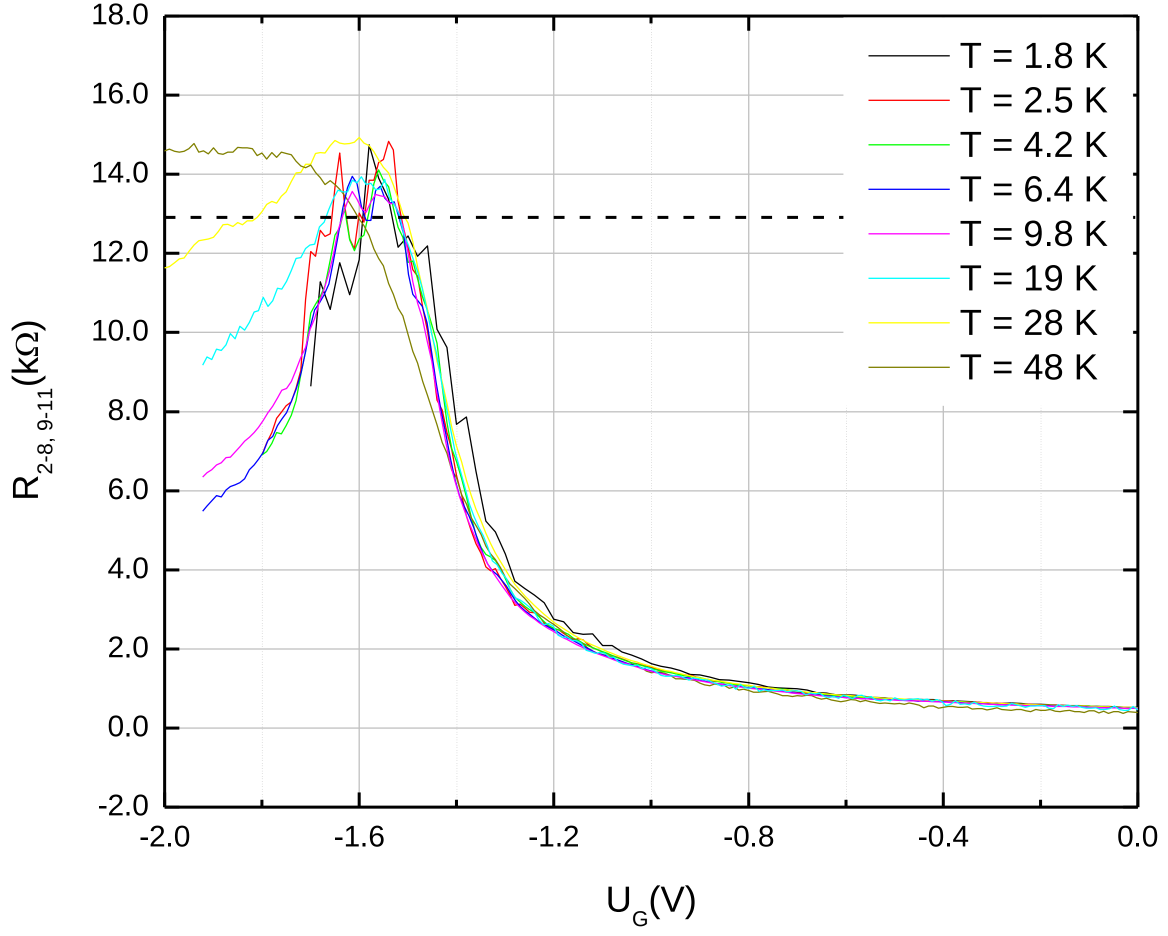
QSHE Size Dependence



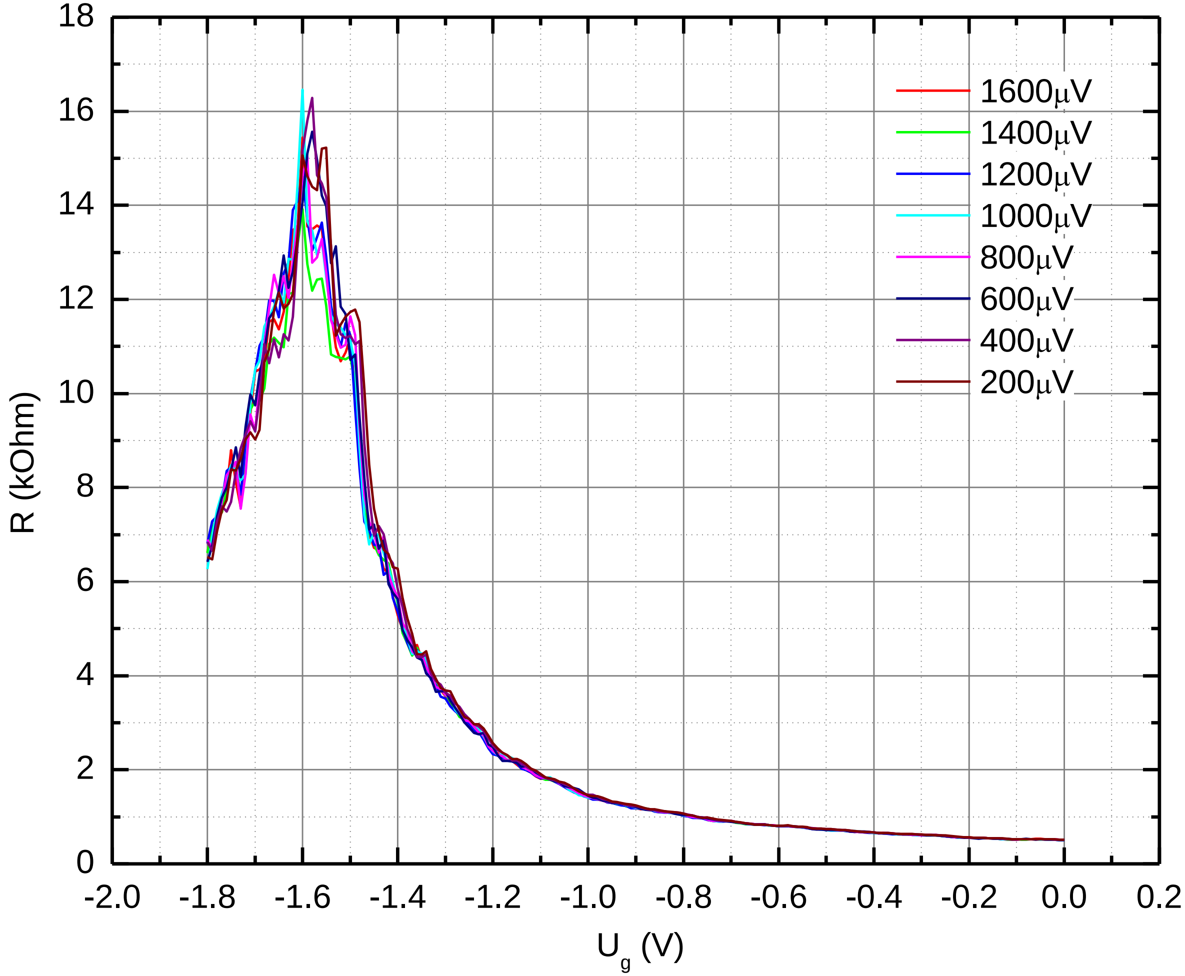
König *et al.*, Science **318**, 766 (2007)

QSHE Temperature Dependence

Q2367 Microhallbar 1 x 1 μm²
I(2-8), U(9-11); B=0 T; T= variabel;

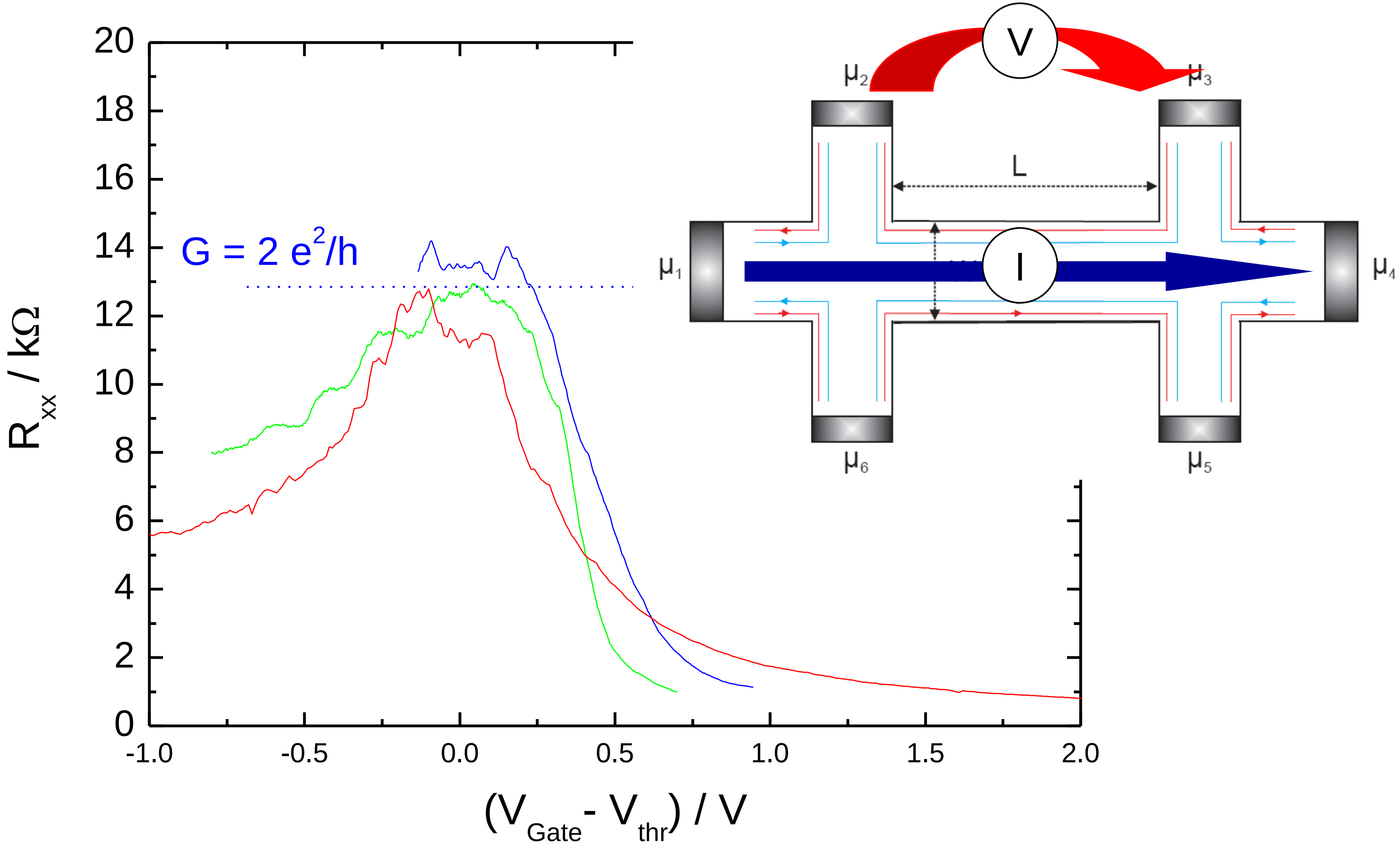


QSHE Excitation Voltage Dependence

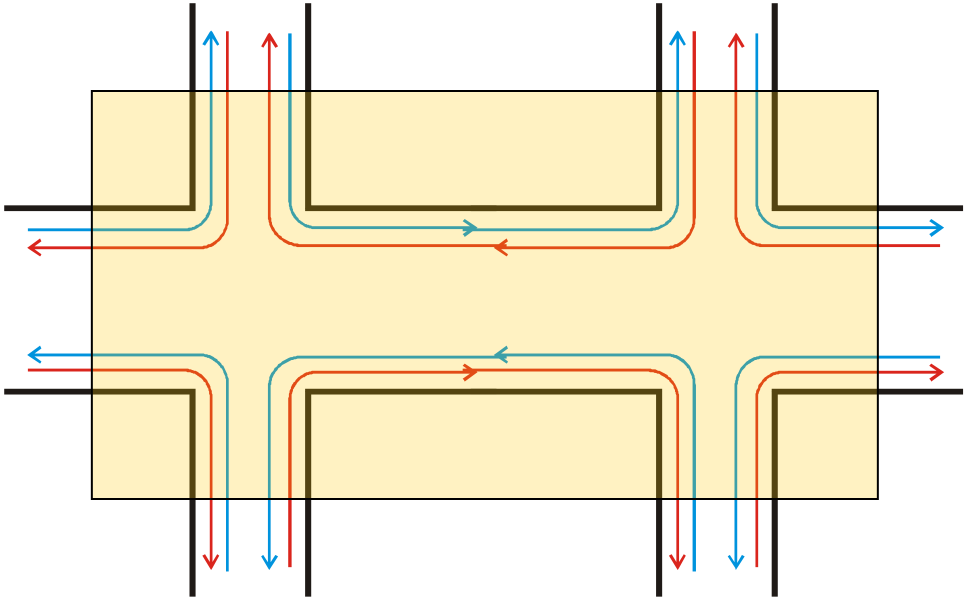


Conductance Quantization

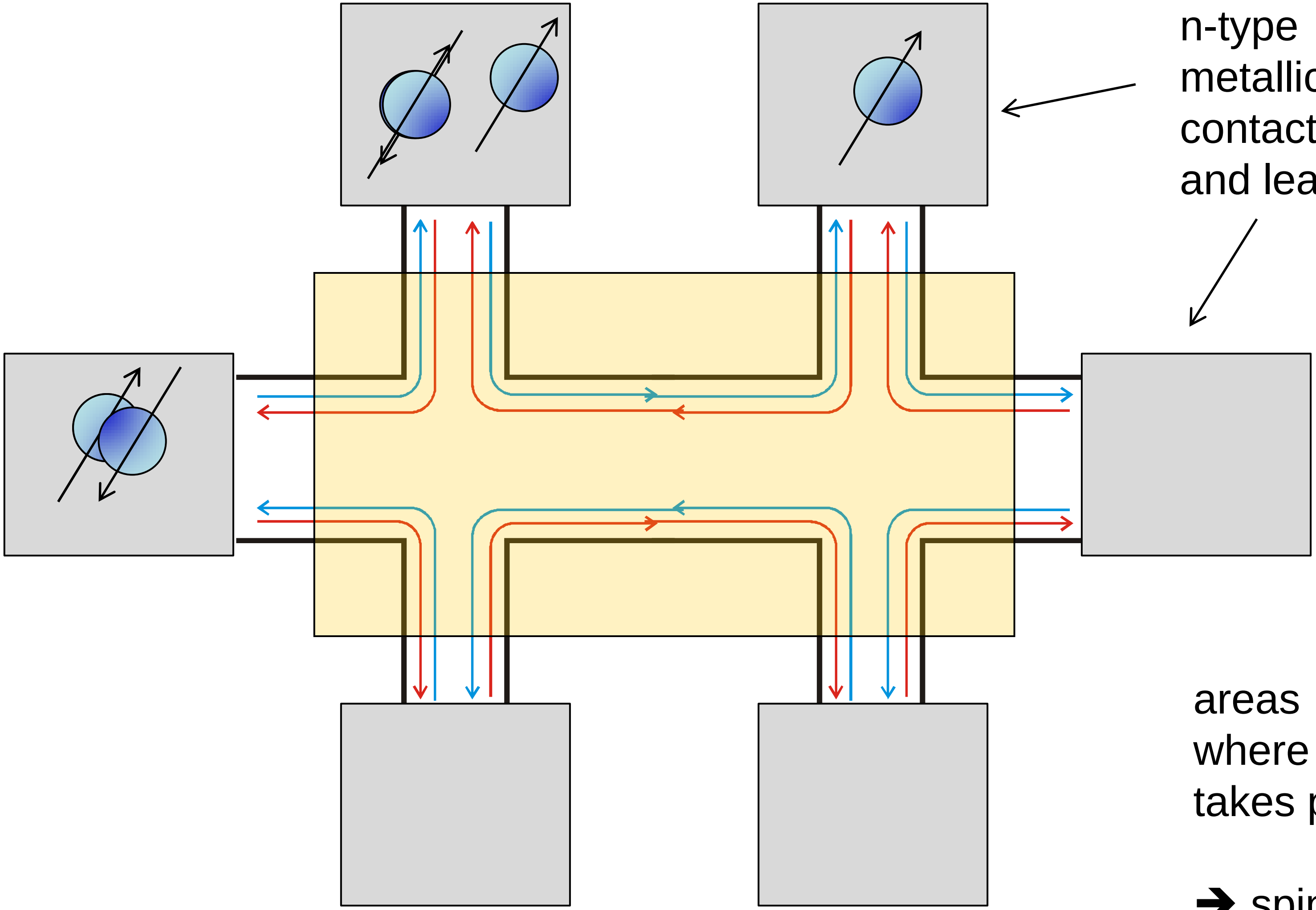
in 4-terminal geometry!?



Small Samples



Role of Ohmic Contacts



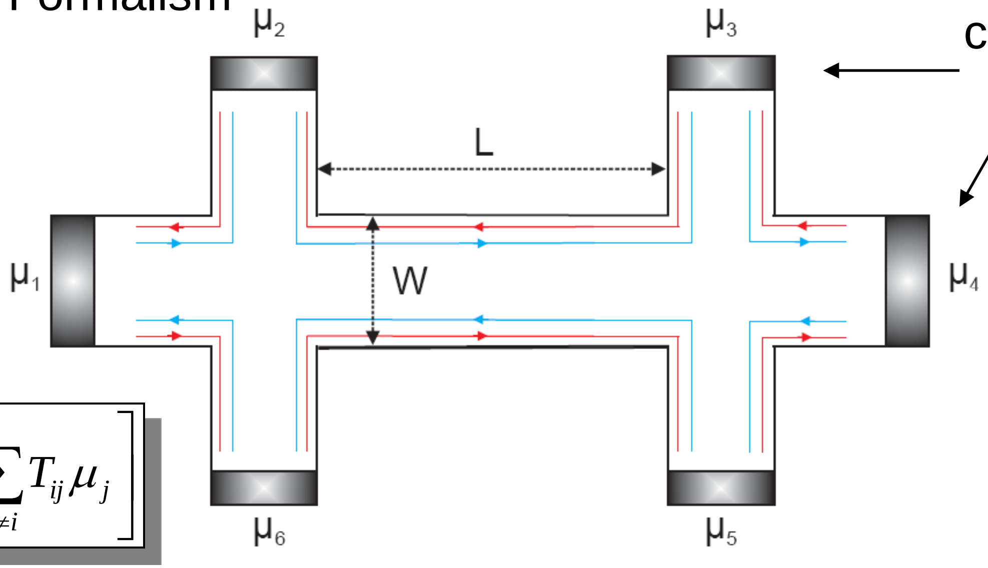
n-type
metallic
contact areas
and leads

areas
where scattering
takes place

➔ spin relaxation

Multi-Terminal Probe

Landauer-Büttiker Formalism



normal conducting contacts
➔ **no QSHE**

$$I_i = \frac{2e}{h}\left[\left(M_i - R_{ii}\right)\mu_i - \sum_{j\neq i} T_{ij}\mu_j\right]$$

$$T = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 1 & 0 & 0 & 0 & 1 & -2 \end{pmatrix} \Rightarrow \begin{cases} G_{4t} = \dfrac{I_{14}}{\mu_3 - \mu_2} = \dfrac{2e^2}{h} \\ G_{2t} = \dfrac{I_{14}}{\mu_4 - \mu_1} = \dfrac{2}{3}\dfrac{e^2}{h} \end{cases}$$

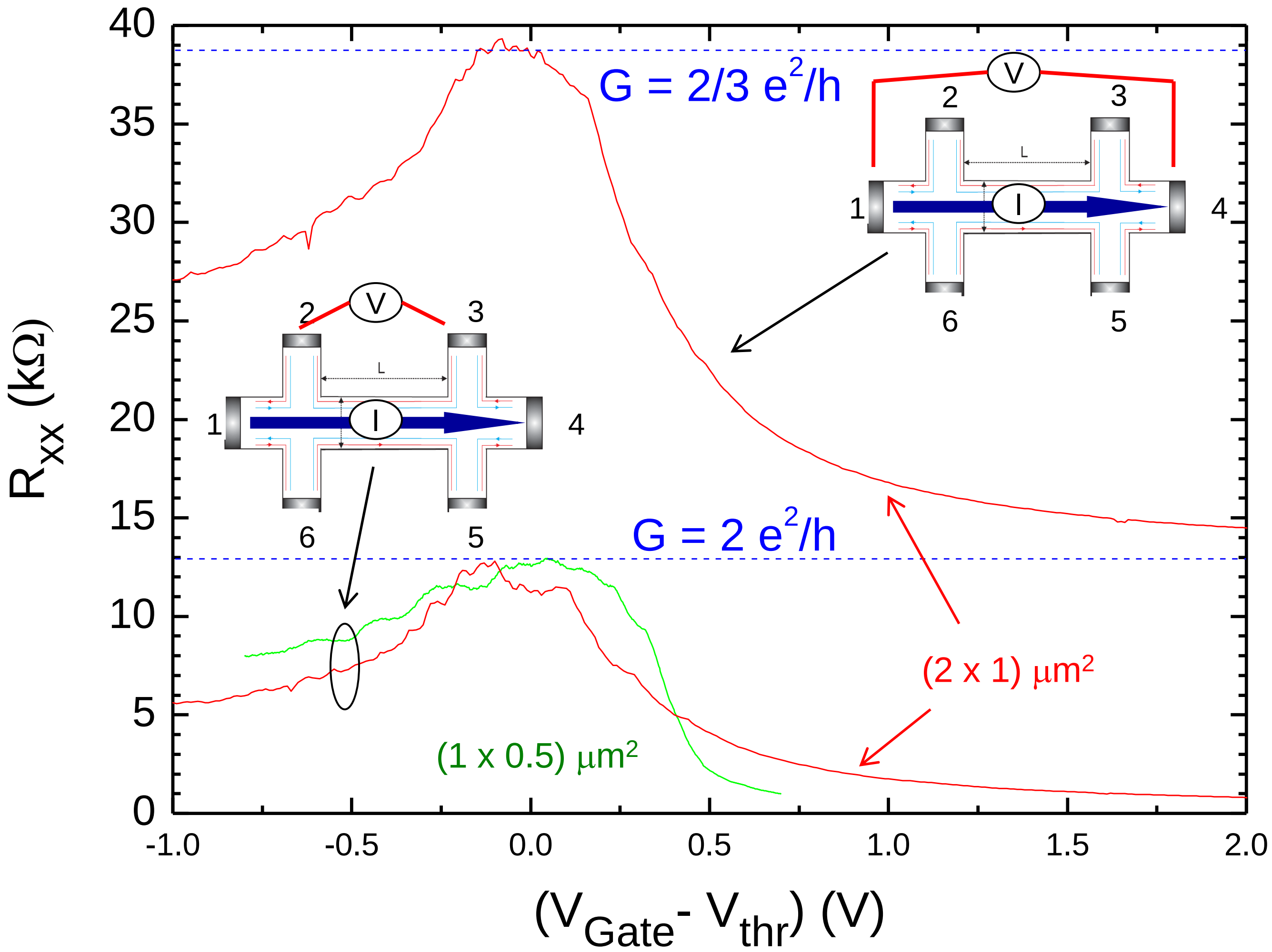
$$G_{4t,\exp} \approx 2\frac{e^2}{h}$$

$$\left.\frac{R_{2t}}{R_{4t}}\right|_{\exp} \approx 3$$

generally $R_{2t} = \dfrac{(n+1)h}{2e^2}$

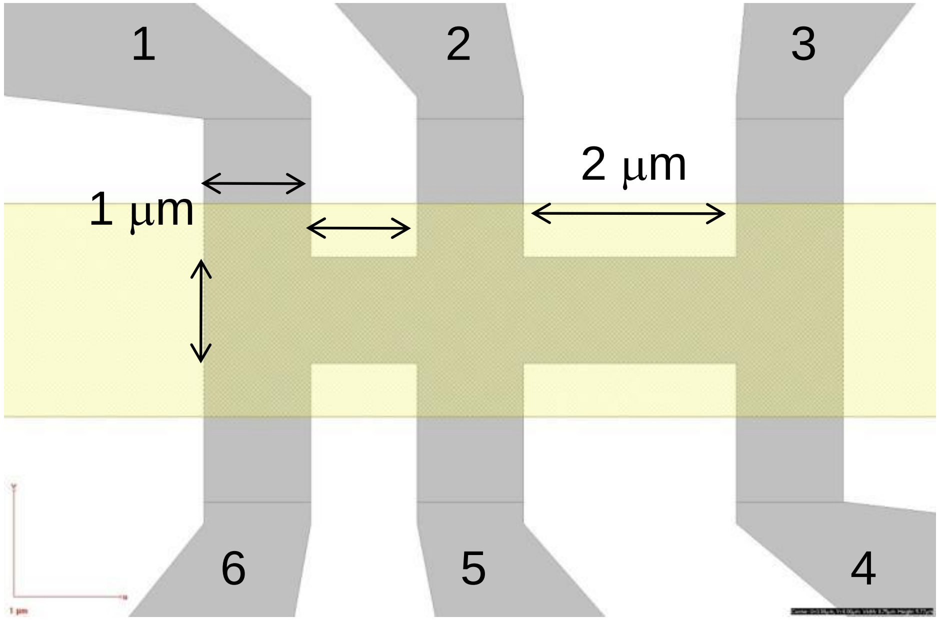
QSHE in inverted HgTe-QWs

EP 3
H. Buhmann



$$\frac{R_{2t}}{R_{4t}} \approx 3$$

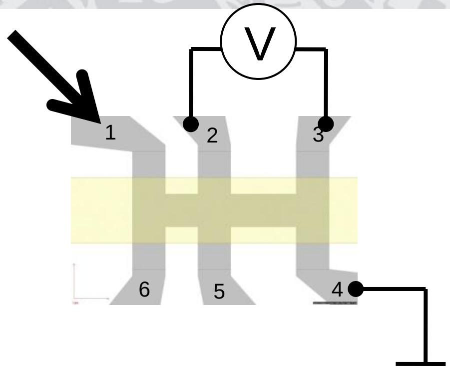
Non-locality

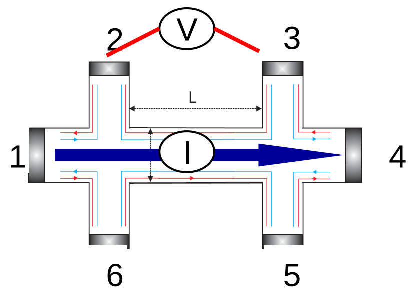


Non-locality

I: 1-4

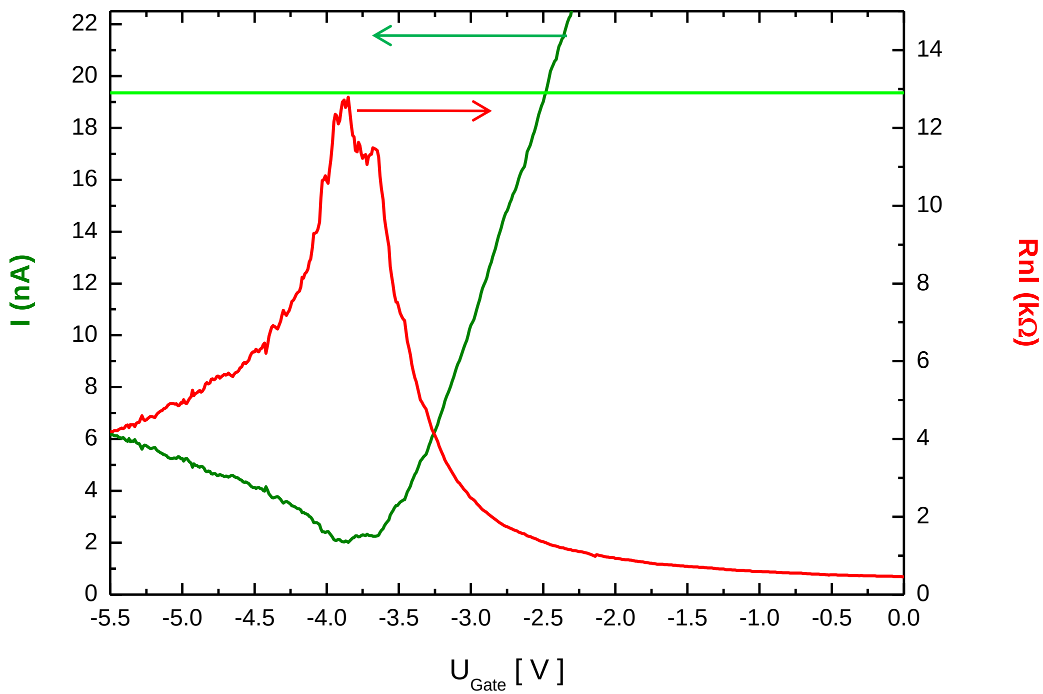
V_{nl}:2-3





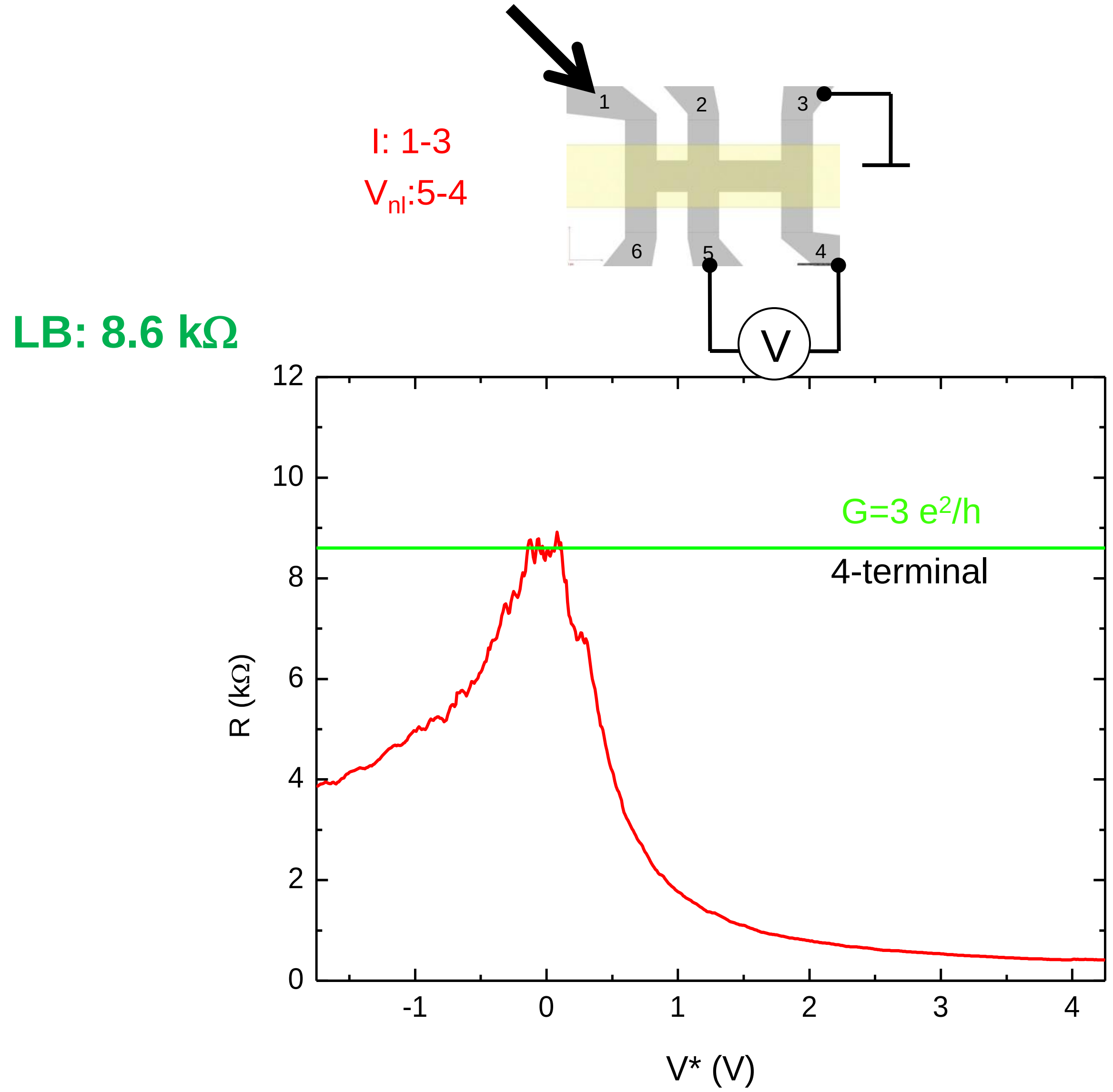
Q2308-02

@6 mK



A. Roth, HB et al.,
Science 325,295 (2009)

Non-locality



A. Roth, HB et al., Science 325,295 (2009)

Non-locality

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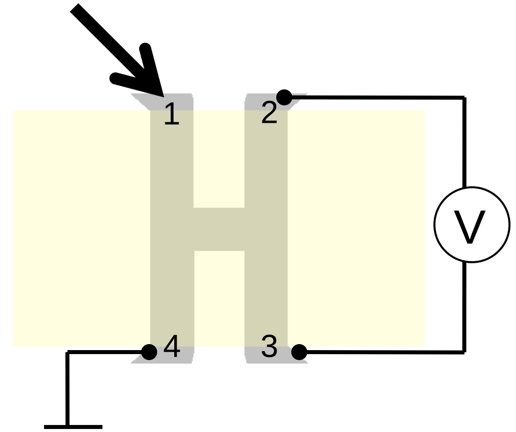
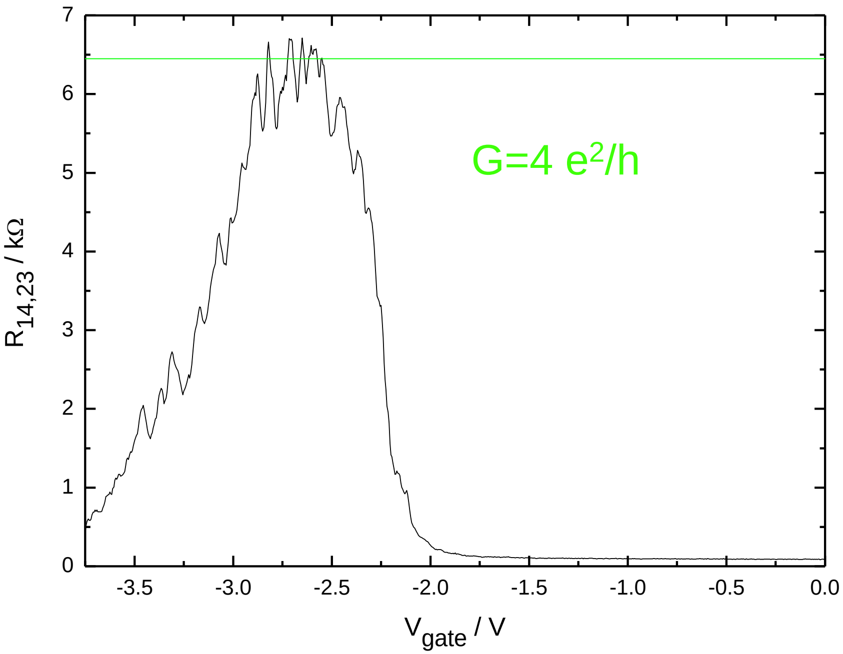
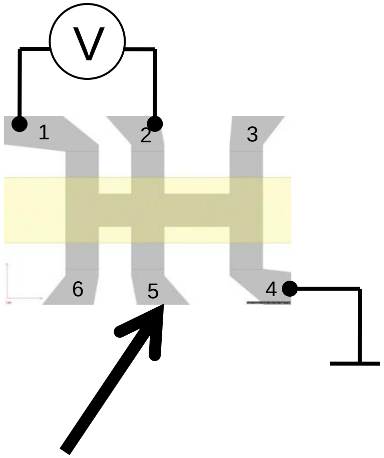




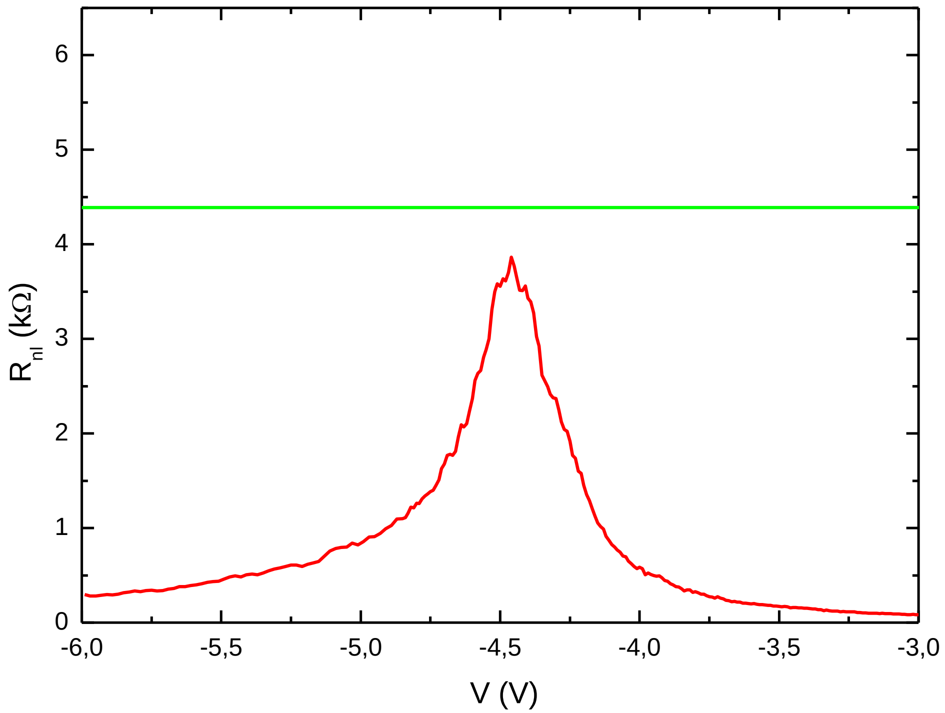
LB: 6.4 kΩ

A. Roth, HB et al.,
Science 325,295 (2009)

Non-locality



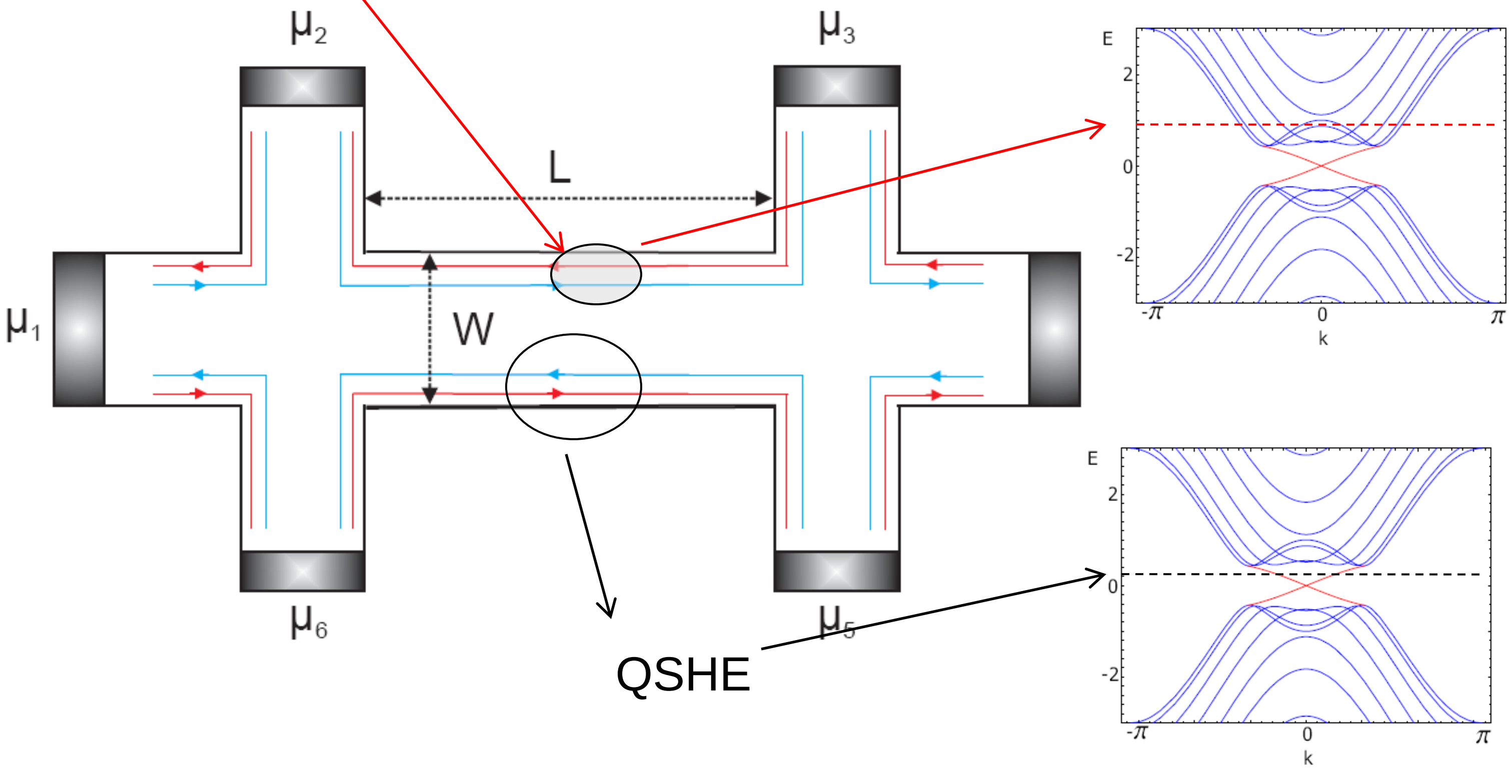
LB: 4.3 kΩ



A. Roth, HB et al.,
Science 325,295 (2009)

Back Scattering

potential fluctuations introduce areas of normal metallic (n- or p-) conductance in which back scattering becomes possible

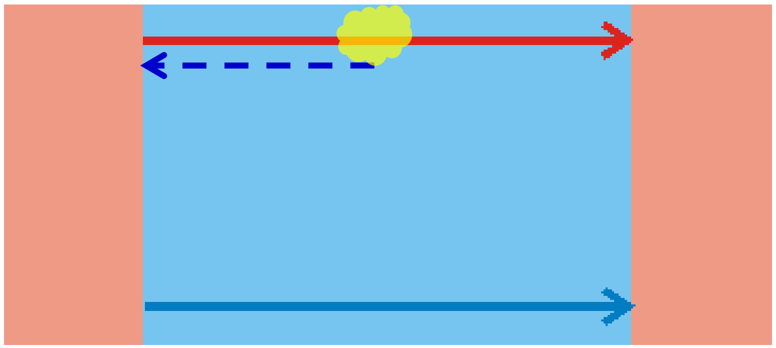


The potential landscape is modified by gate (density) sweeps!

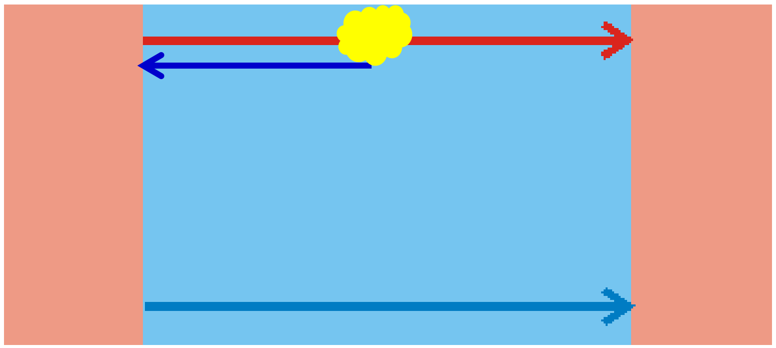
Back Scattering

area of raised or lowered chem. Pot.

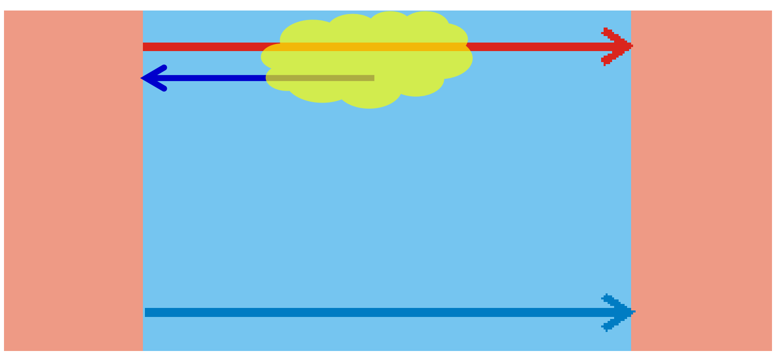
the probalility for back scatterring increases



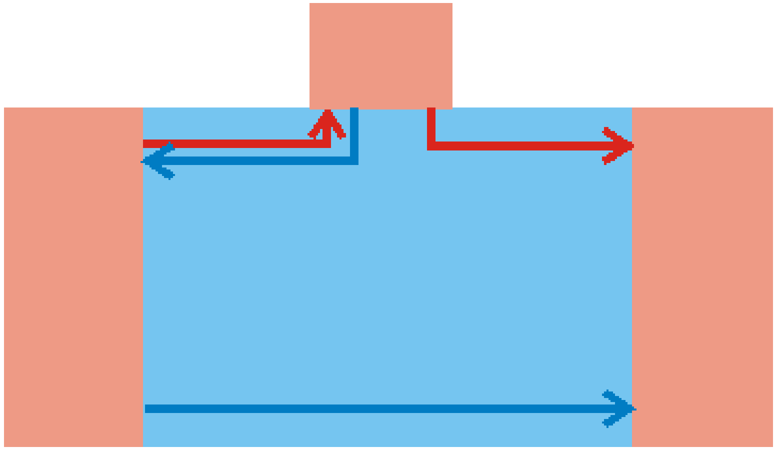
strength



length



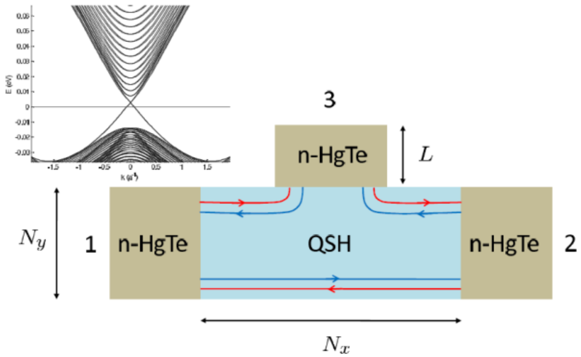
➔ additional ohmic contact

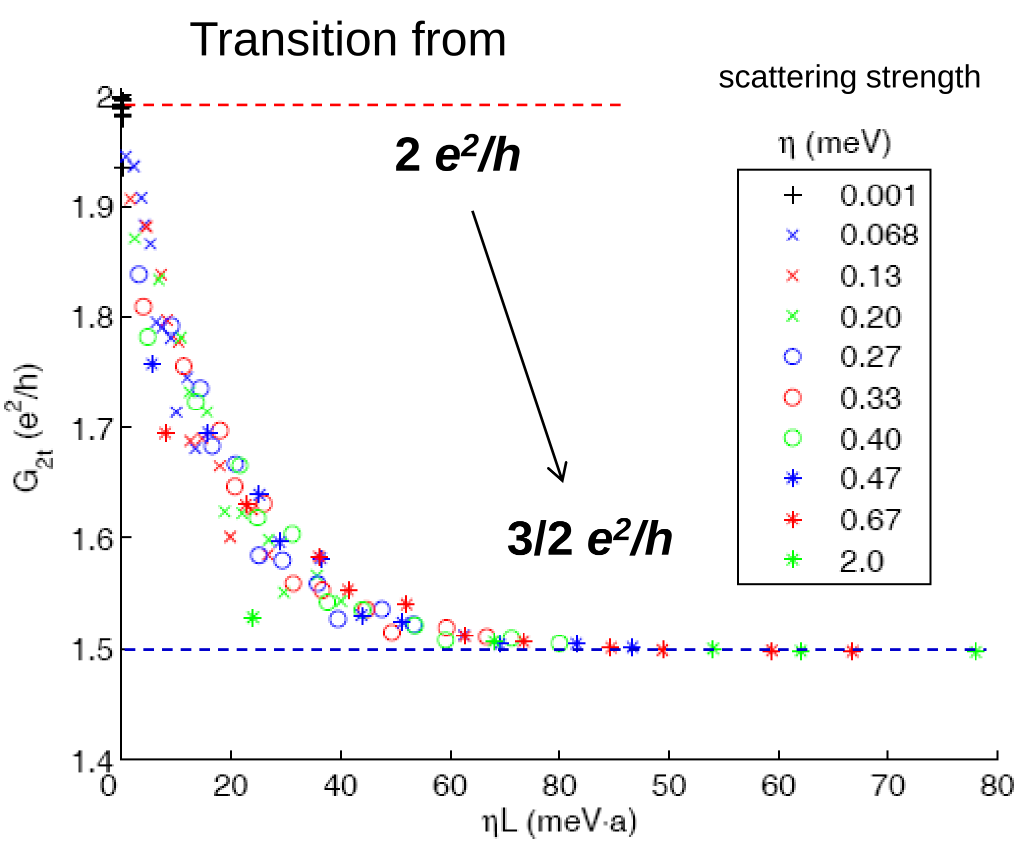


transition from a two to a three terminal device

Additional Scattering Potential

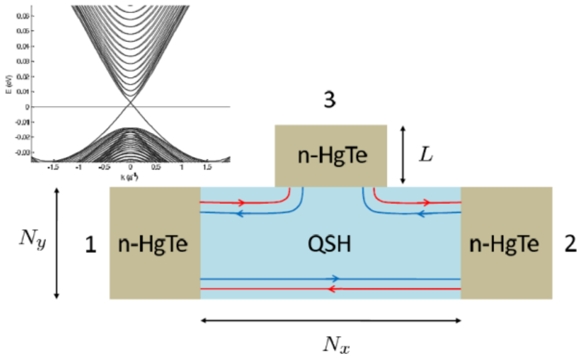
H. Buhmann

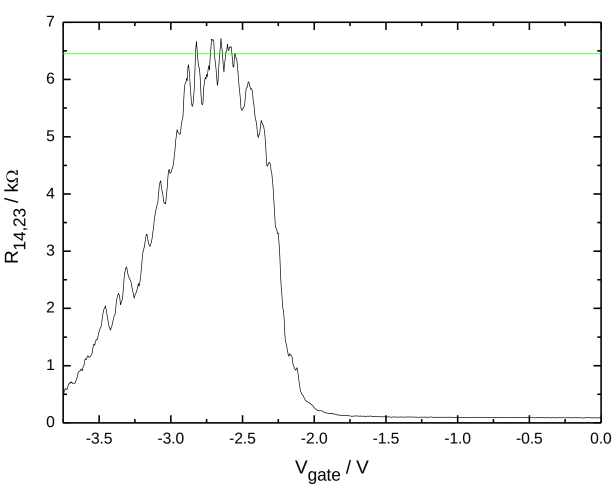


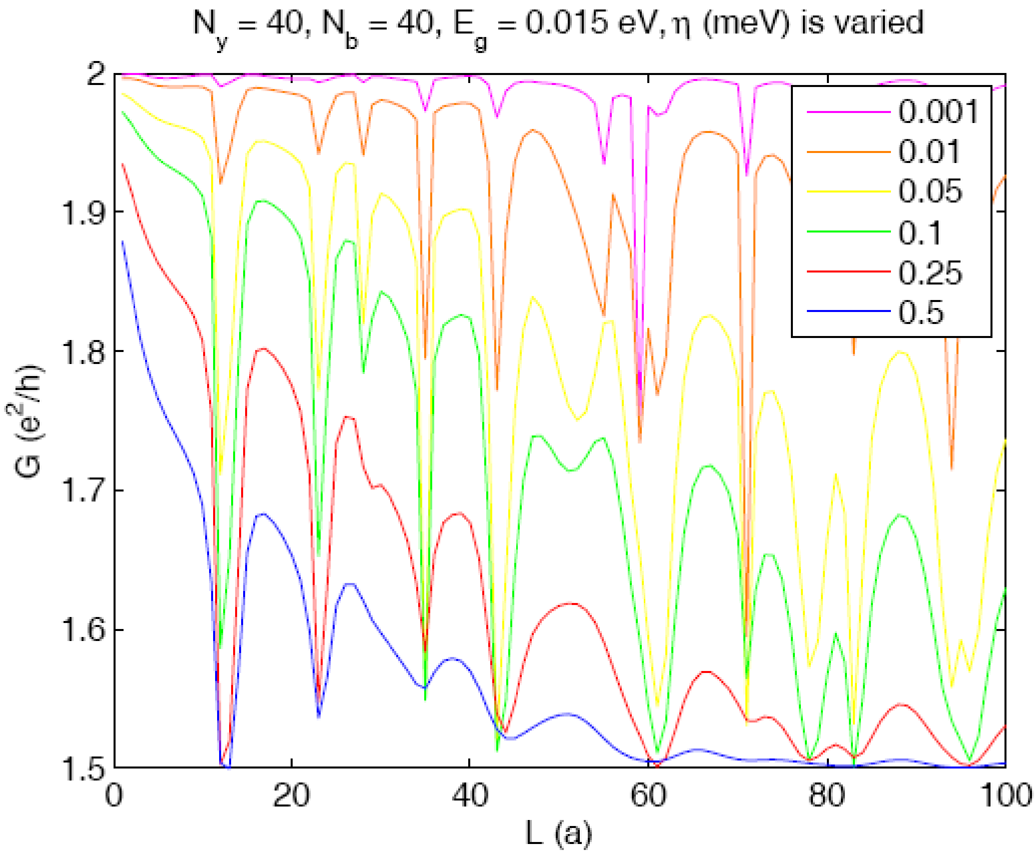


A. Roth, HB et al.,
Science 325,295 (2009)

Additional Scattering Potential

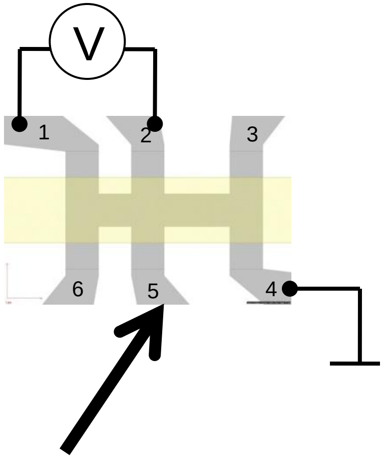






A. Roth, HB et al.,
Science 325,295 (2009)

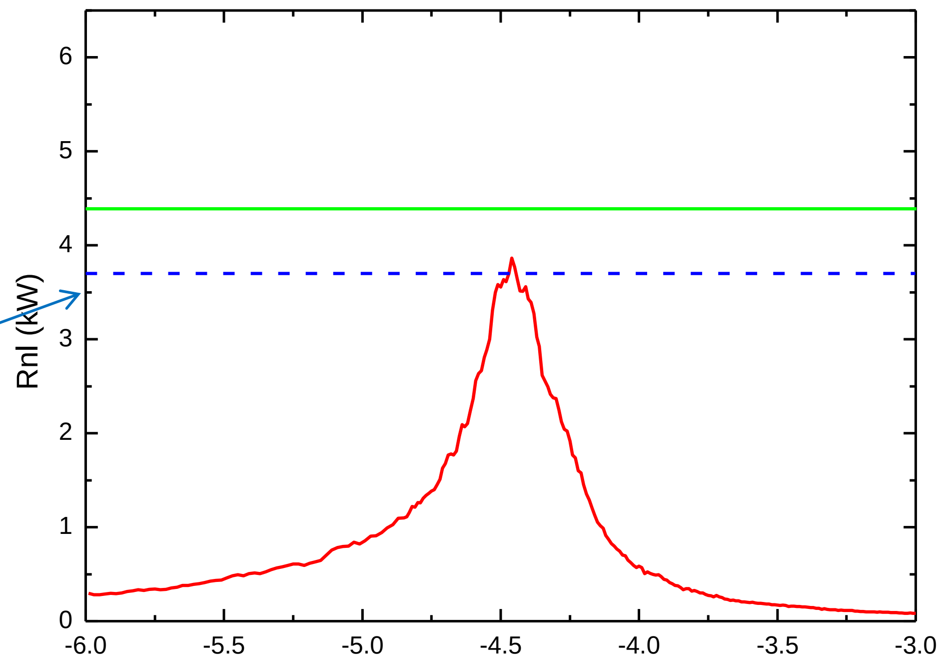
Potential Fluctuations

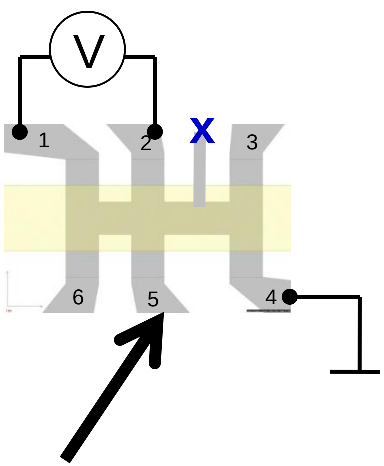


LB: 4.3 kΩ

one additional contact:
- between 2 and 3

➔ **3.7 kΩ**



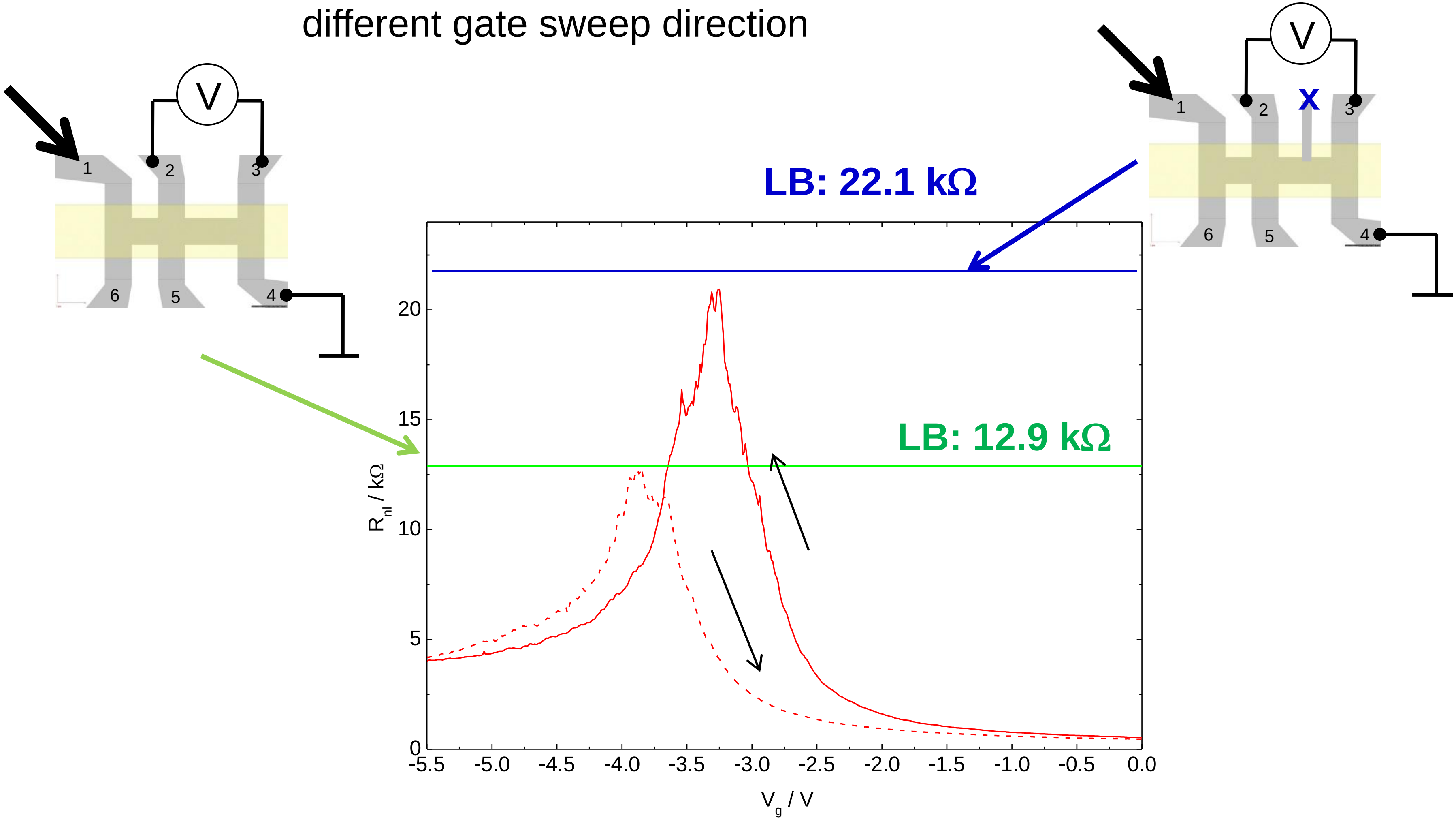


A. Roth, HB et al.,
Science 325,295 (2009)

Potential Fluctuations

H. Buhmann

different gate sweep direction



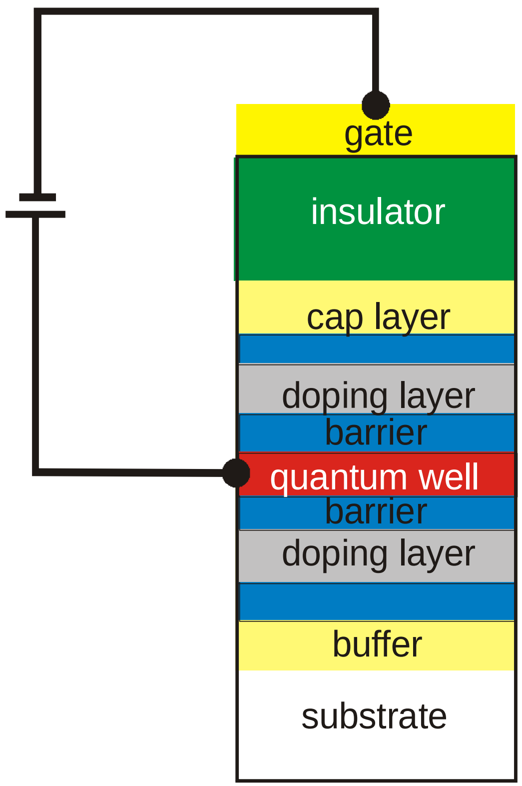
• Hysteresis effects due to charging of trap states at the SC-insulator interface

J. Hinz, HB *et al.*, Semicond. Sci. Technol. **21 (2006) 501–506**

HgTe Quantum Well Structures

H. Buhmann

layer structure



Au

100 nm Si_3N_4/SiO_2

25 nm HgCdTe x = 0.7

10 nm HgCdTe x = 0.7

9 nm HgCdTe with I

10 nm HgCdTe x = 0.7

4 - 12 nm HgTe

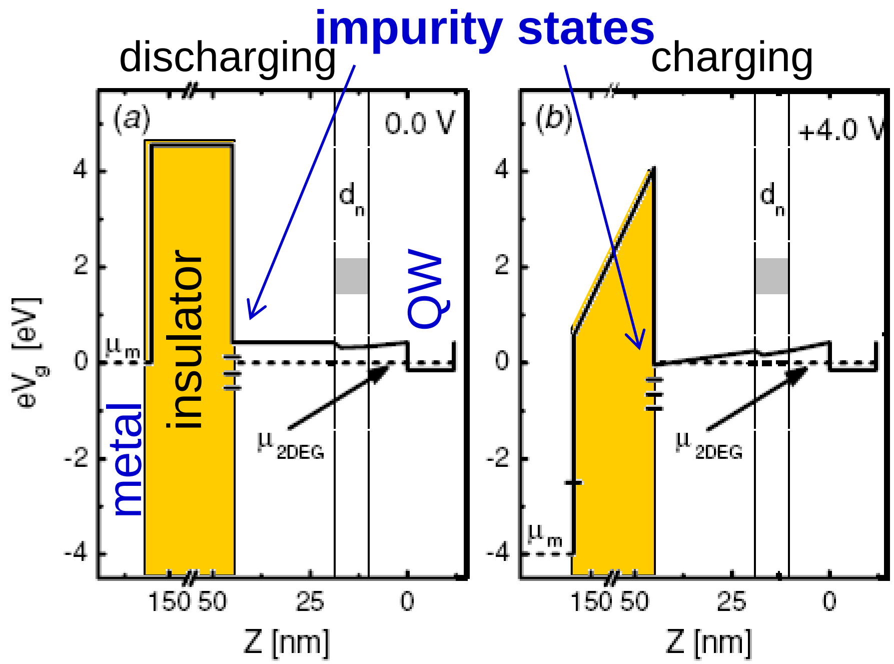
10 nm HgCdTe x = 0.7

9 nm HgCdTe with I

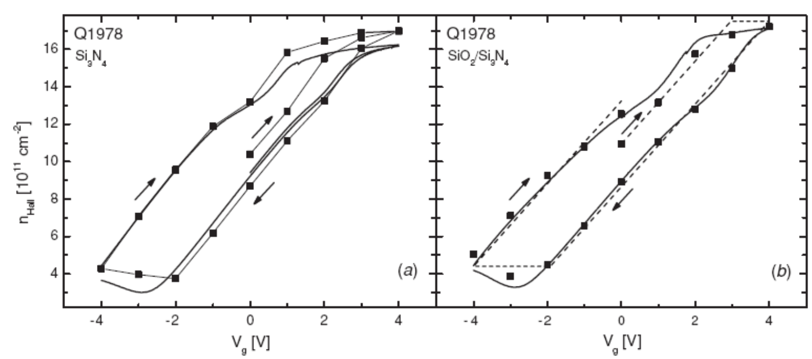
10 nm HgCdTe x = 0.7

25 nm CdTe

CdZnTe(001)

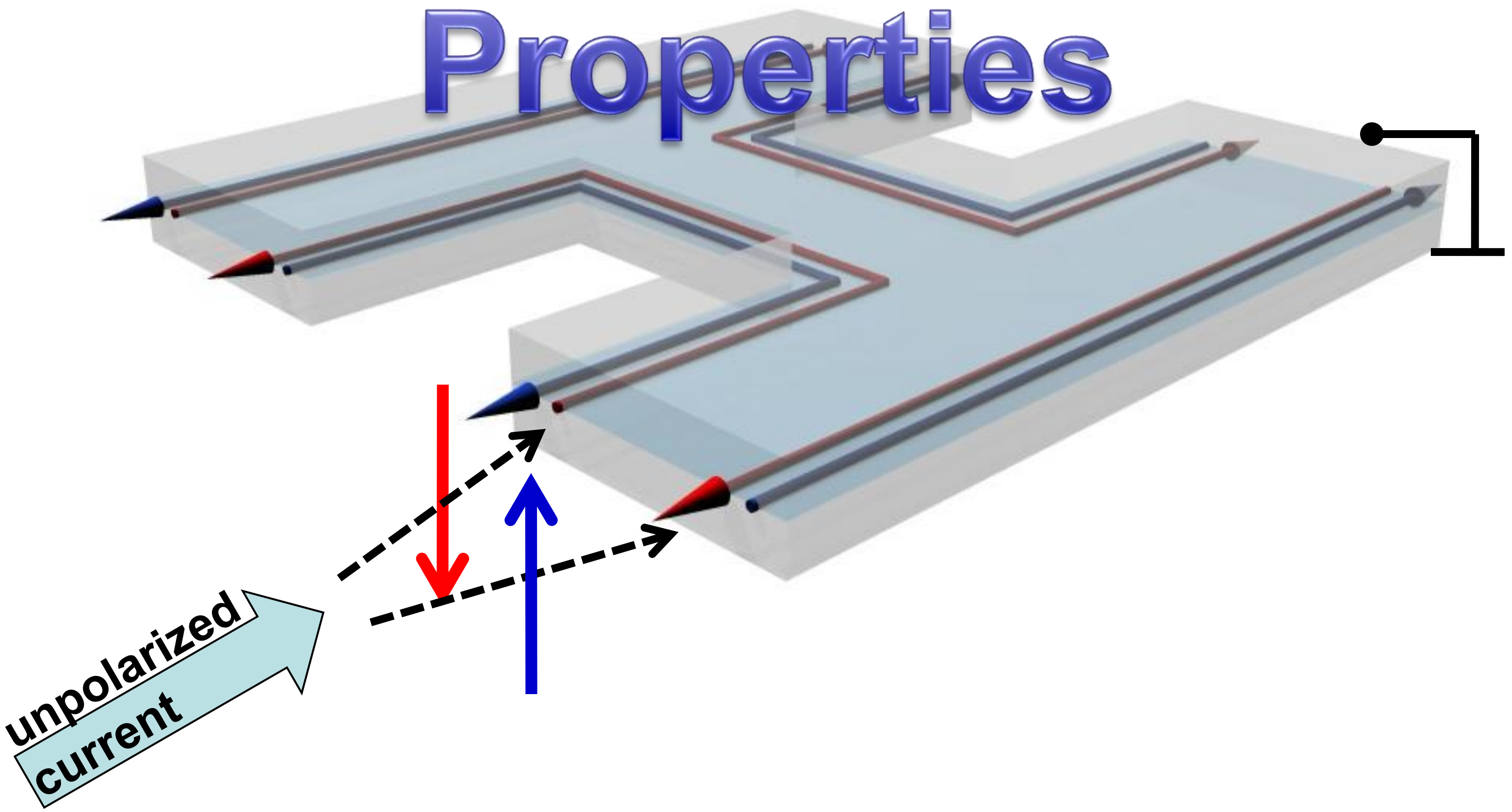


hysteresis effects:

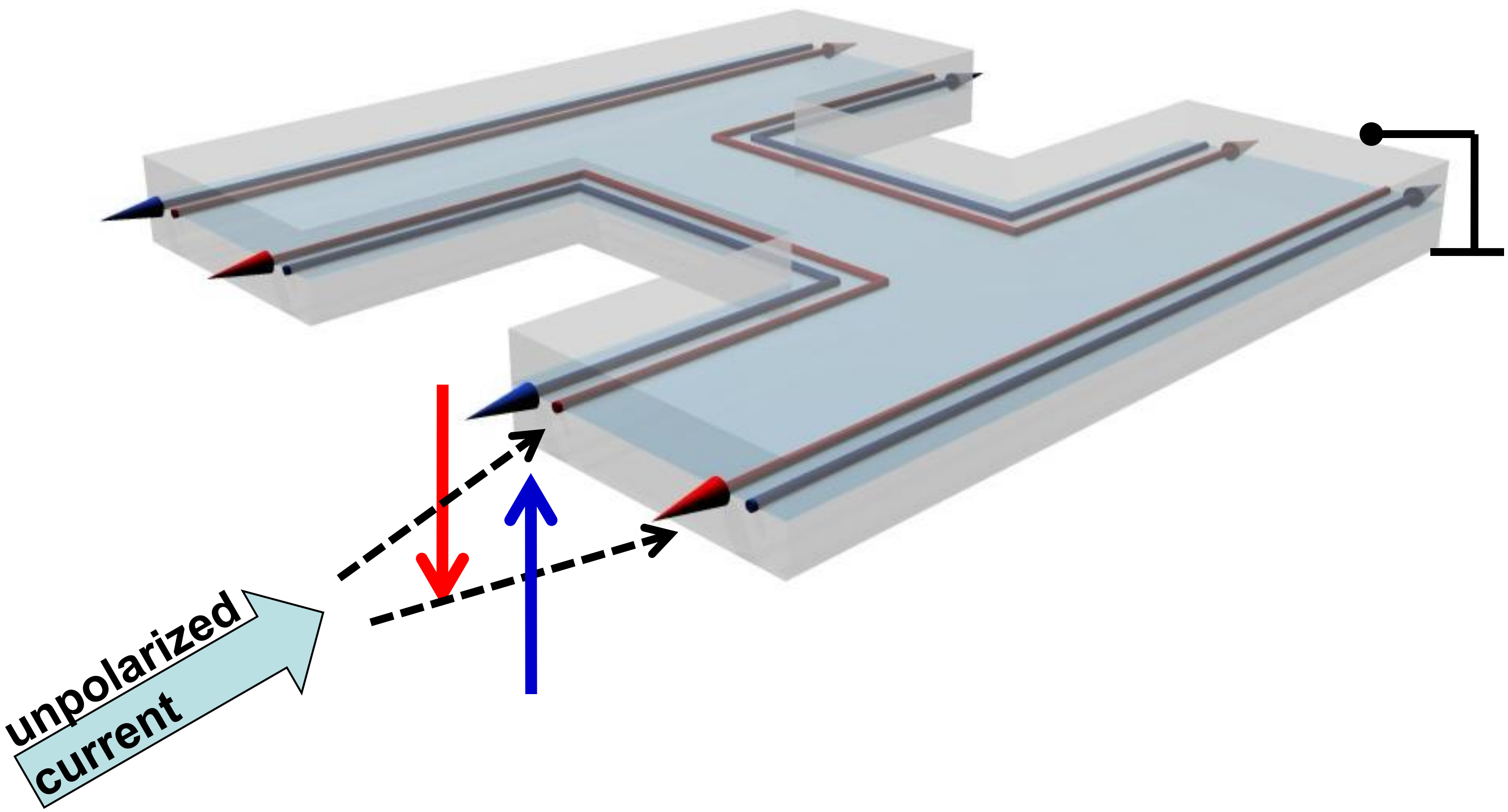


J. Hinz *et al.*, Semicond. Sci. Technol. **21 (2006) 501–506**

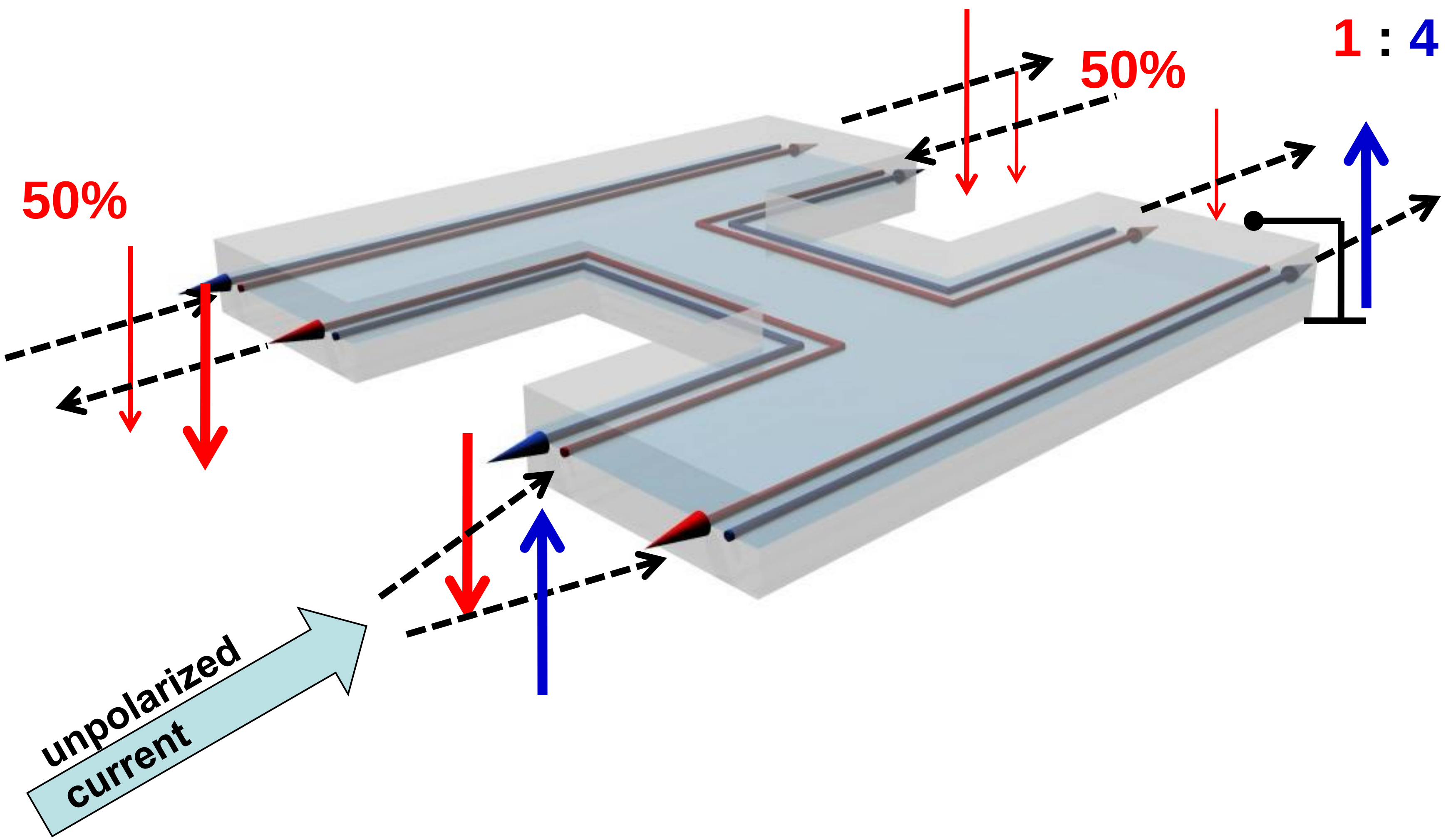
Spin Polarizing Properties



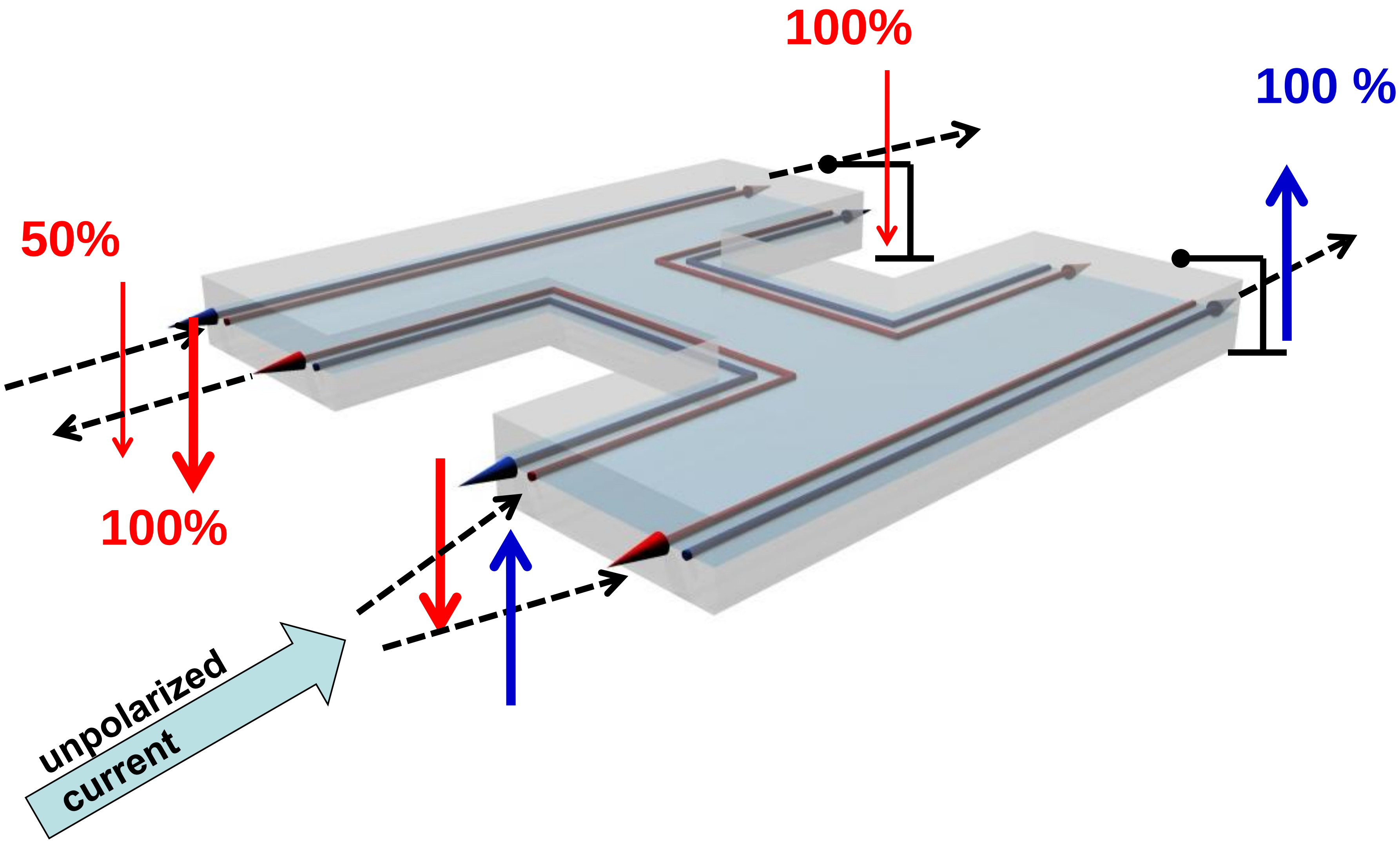
Spin Polarizer



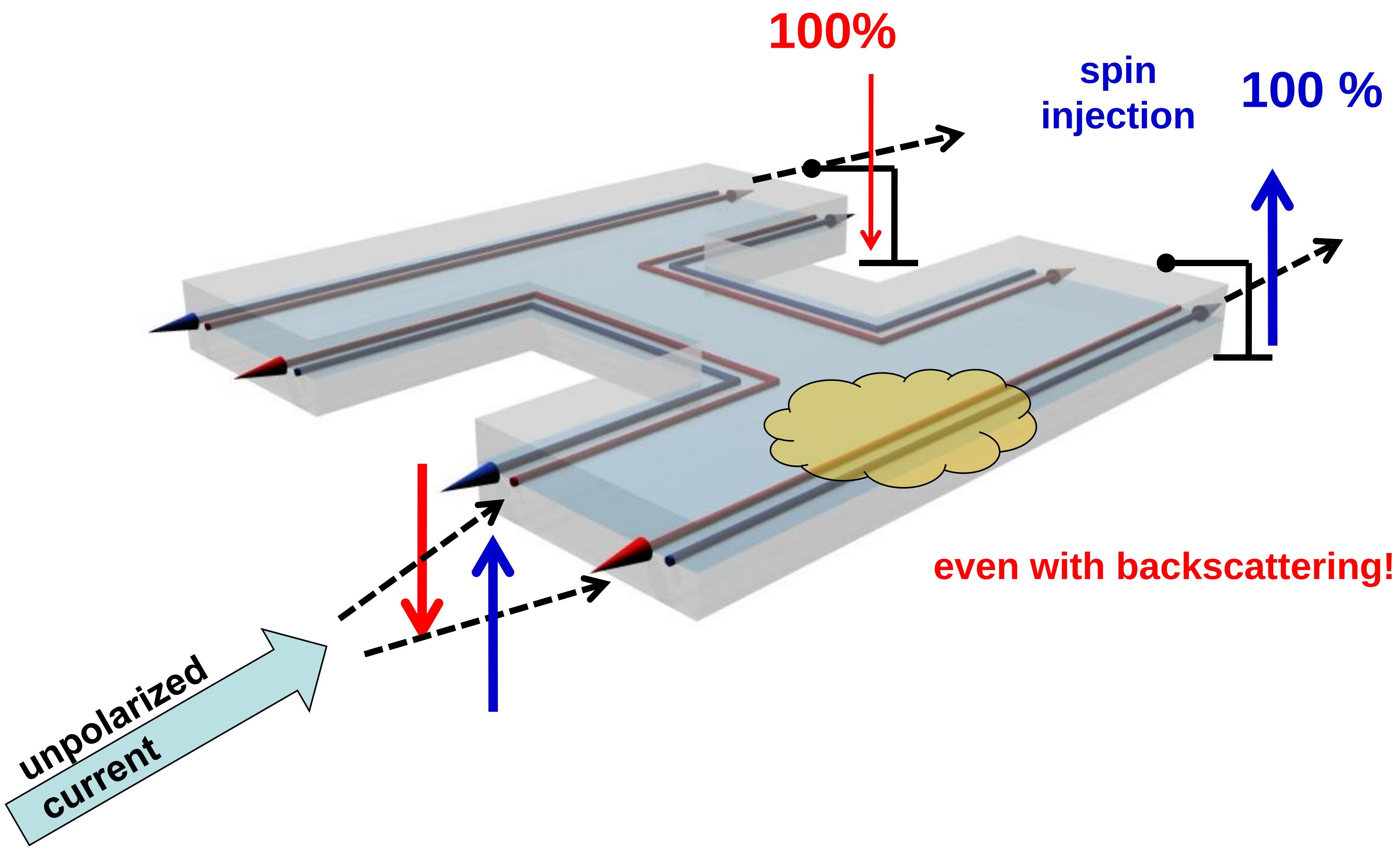
Spin Polarizer



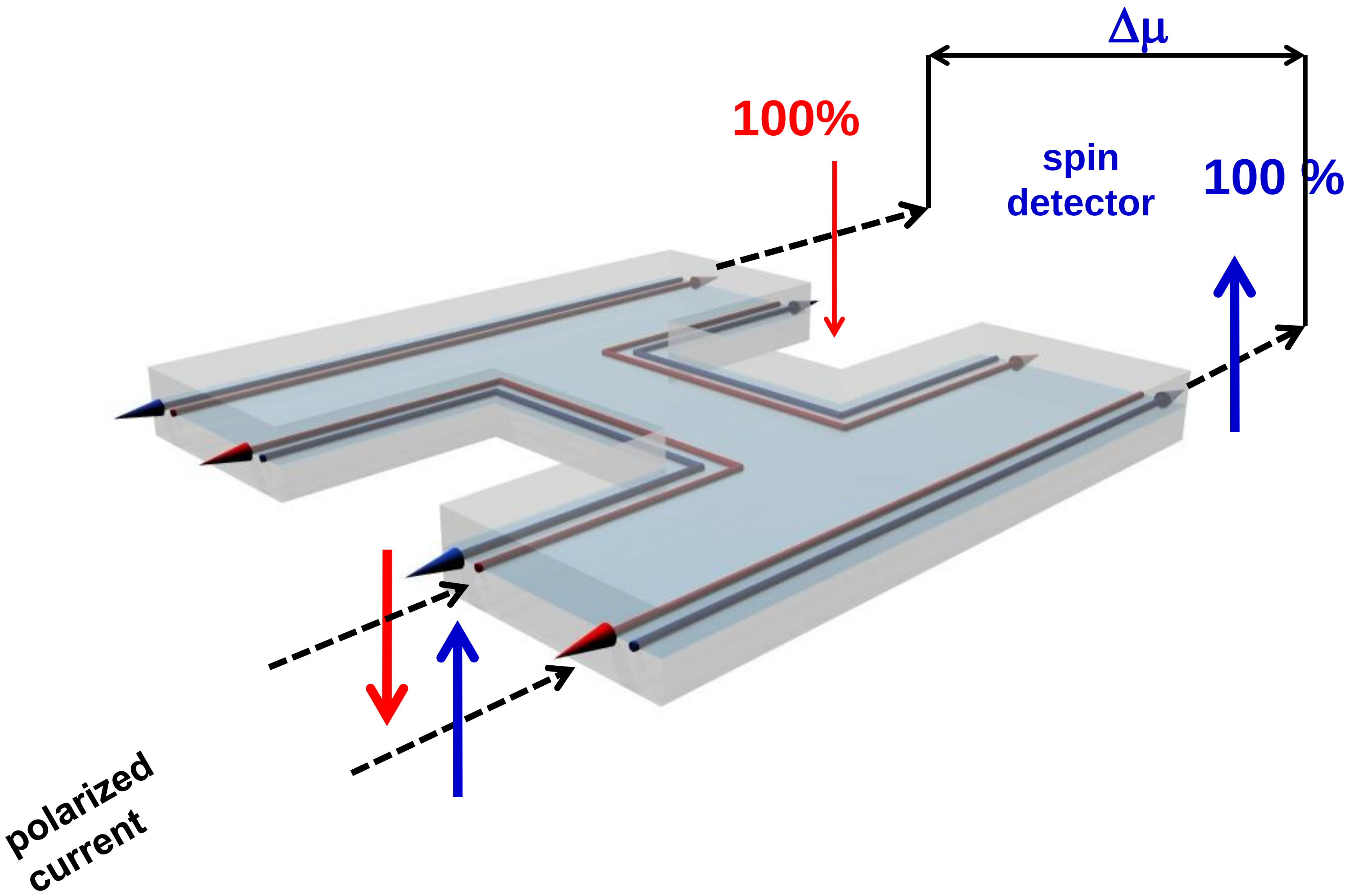
Spin Polarizer



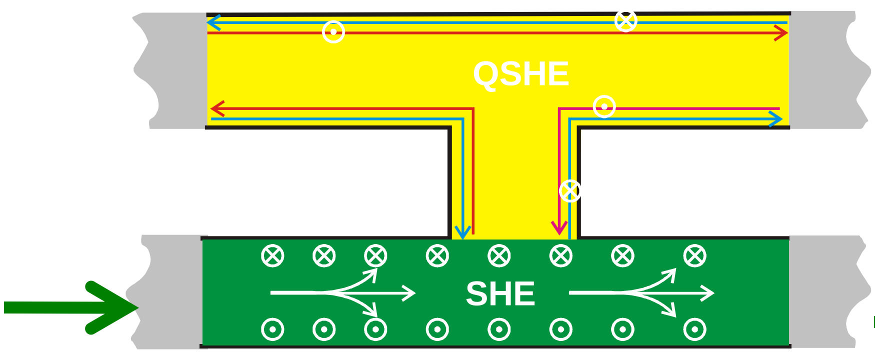
Spin Injector

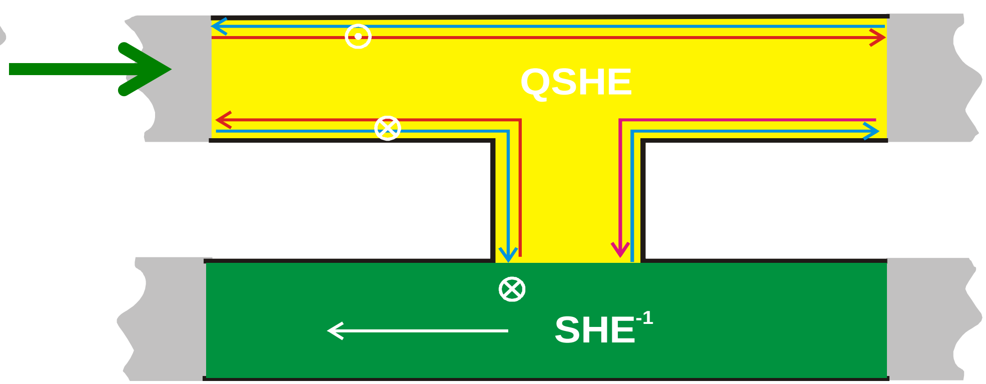


Spin Detection



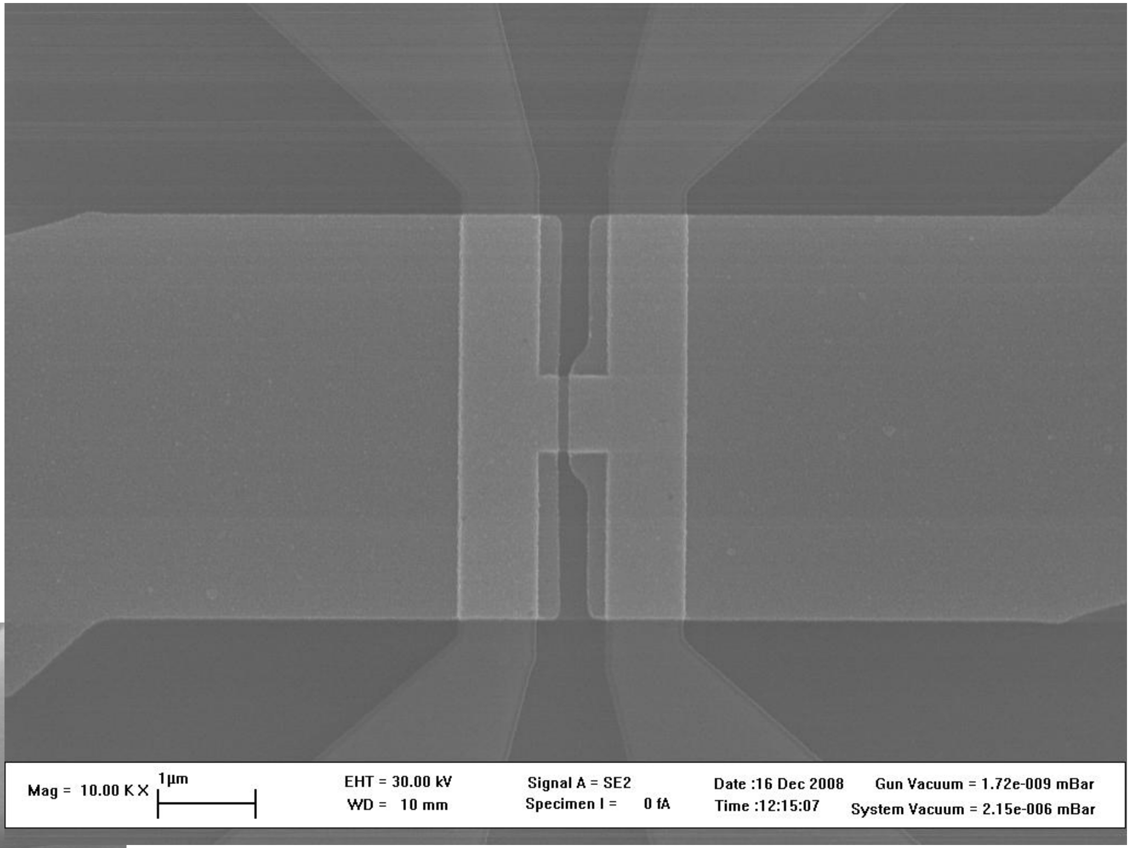
QSHE as spin detector and injector

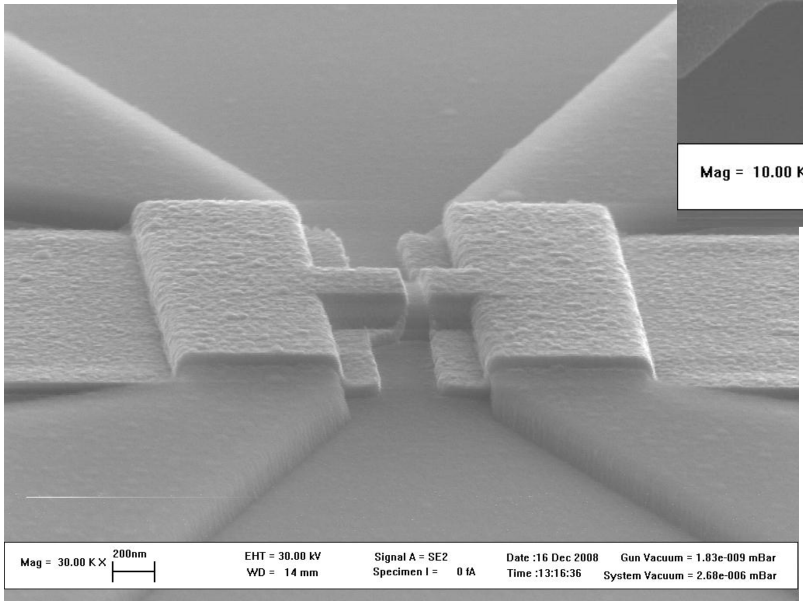




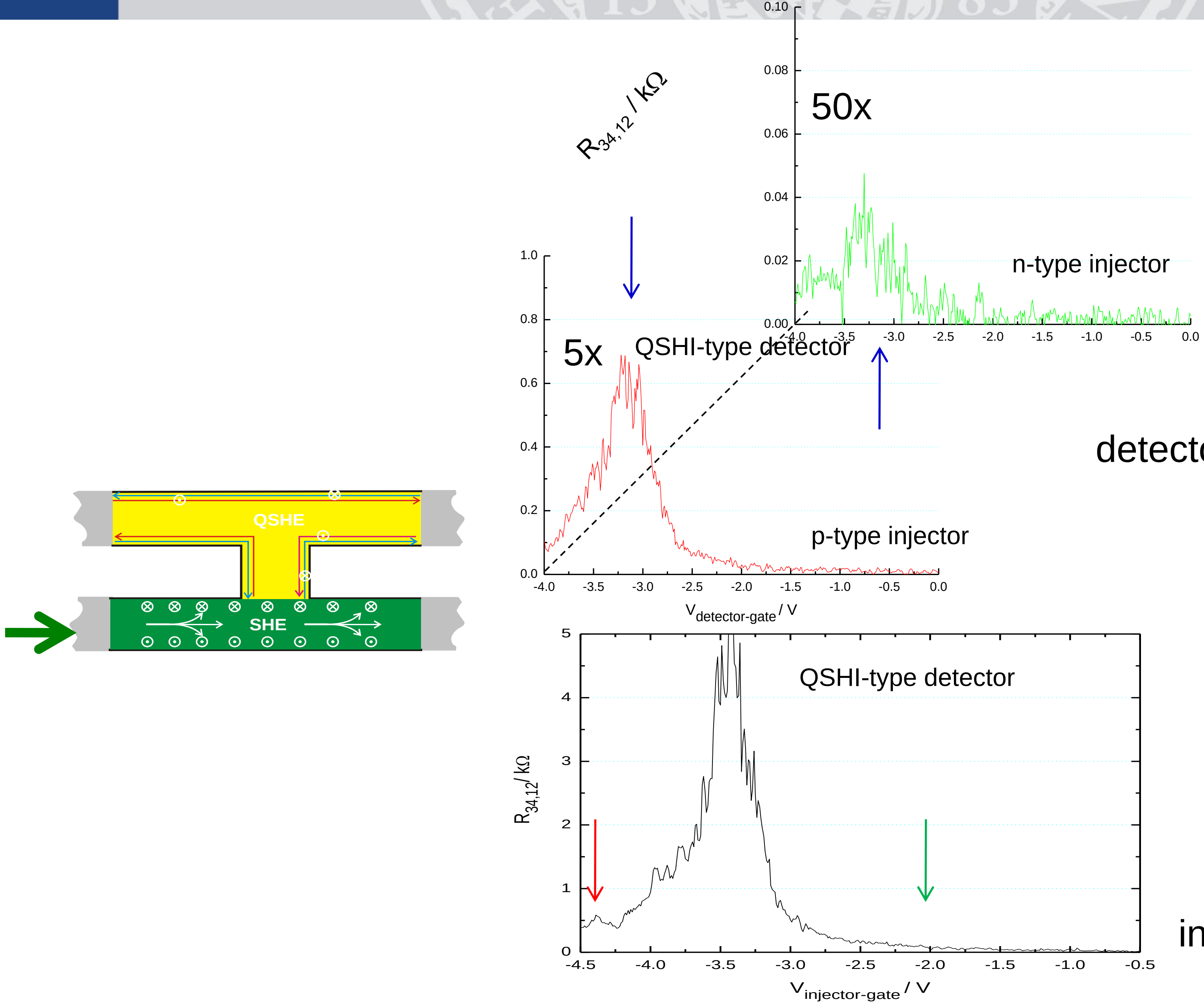
Spin Hall Effect
as creator and analyzer of
spin polarized electrons

Split Gate H-Bar





QSHE Spin-Detector

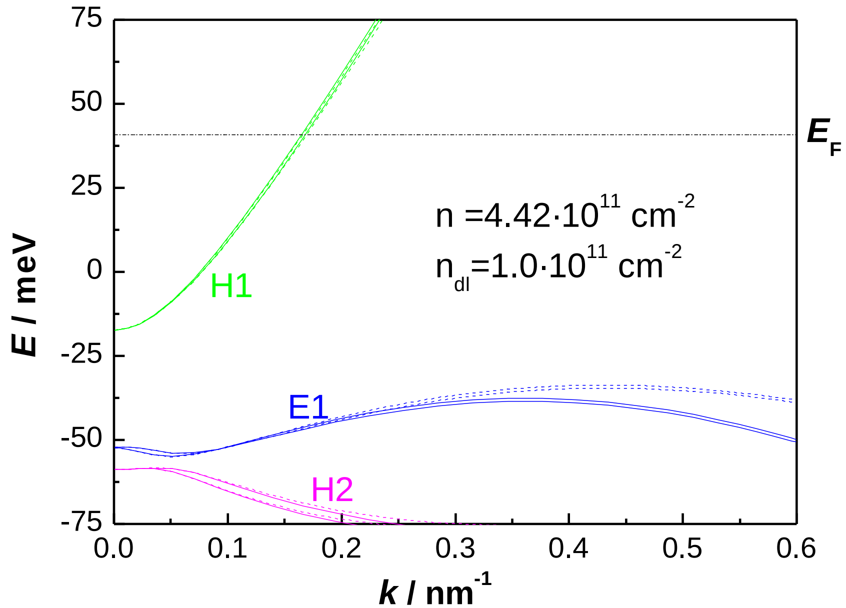


detector sweep

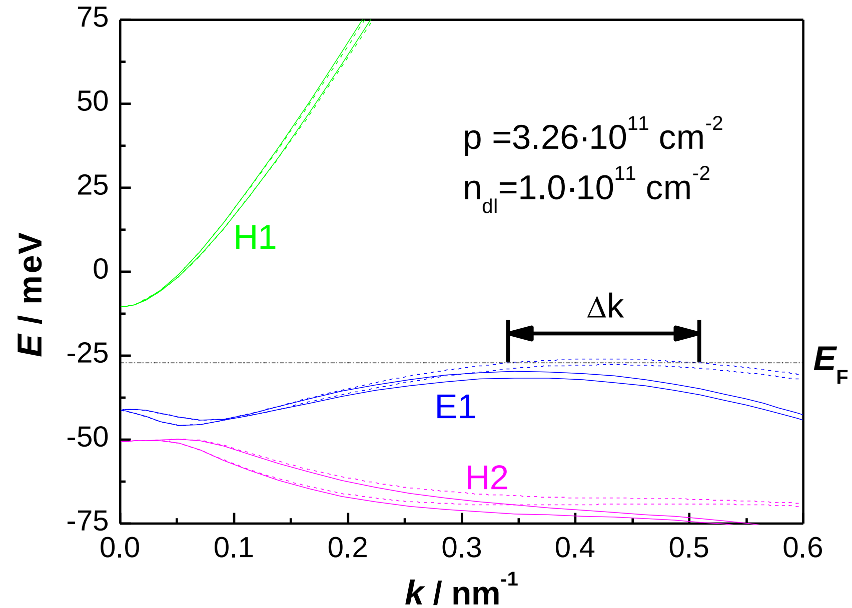
injector sweep

Q2198

n-regime



p-regime



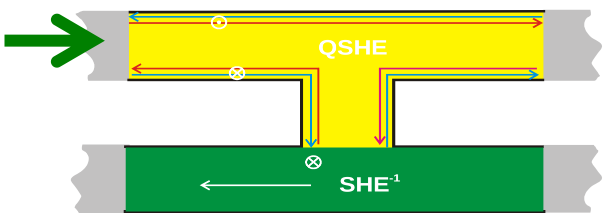
Intrinsic Spin Hall Effekt
Nature Physics 6, 448 (2010)

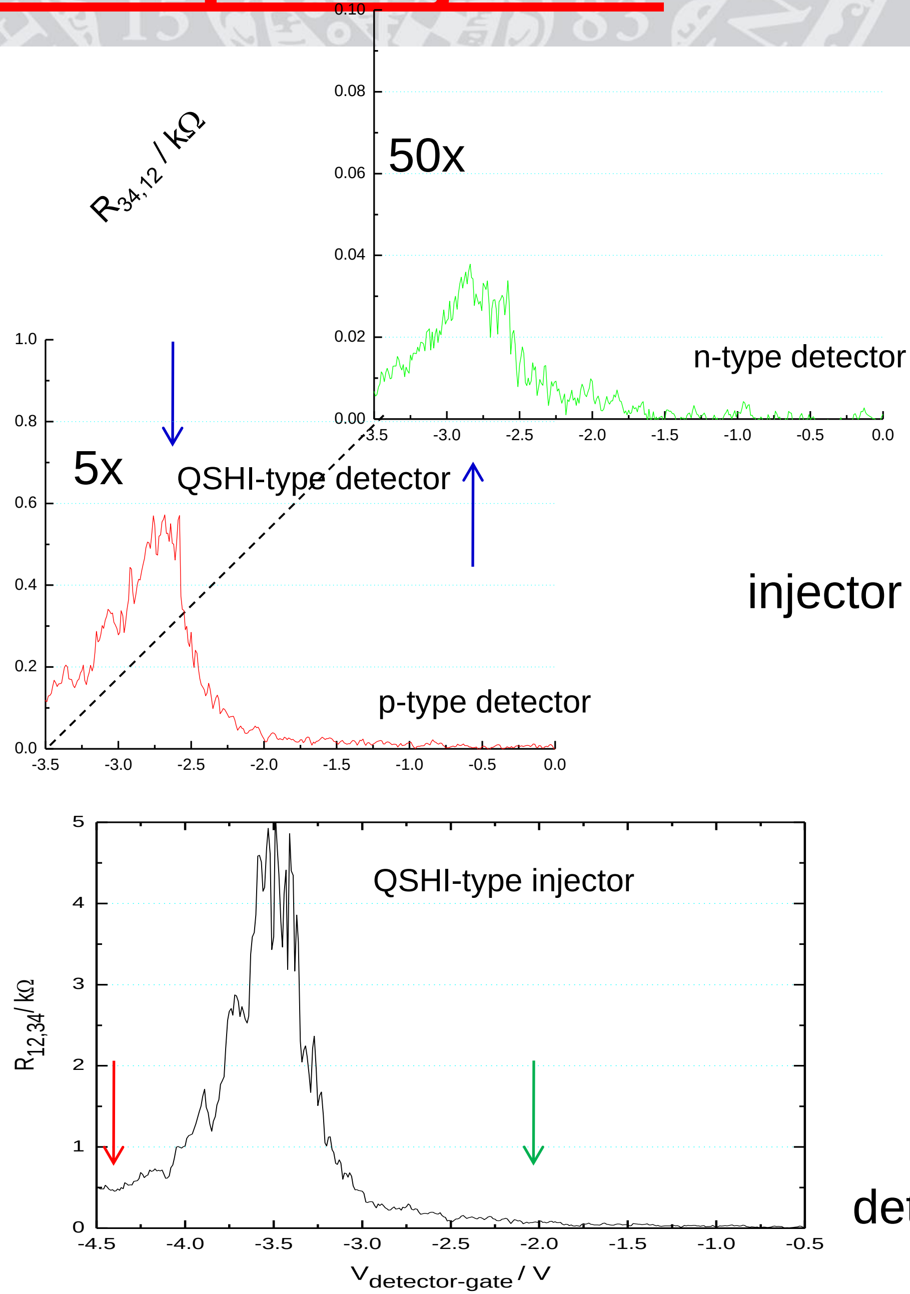
$$j_{s,y} = \frac{-eE_x}{16\pi\lambda m}\left(p_{F+} - p_{F-}\right) \propto \Delta k$$

J.Sinova et al.,
Phys. Rev. Lett. **92**, 126603 (2004)

QSHE Spin-Injector

EP 3
H. Buhmann



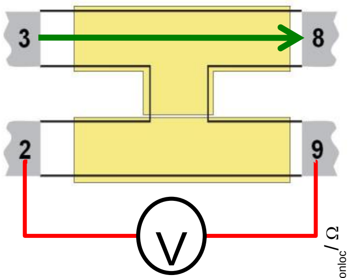


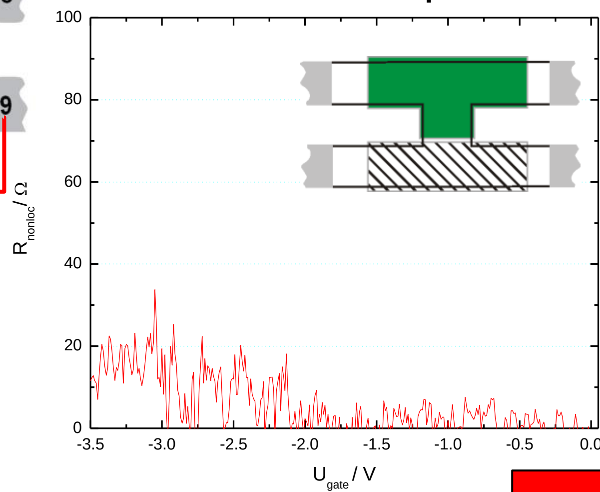
injector sweep

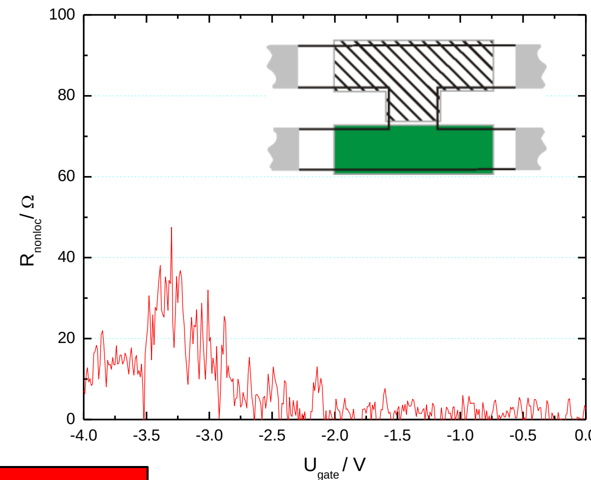
detector sweep

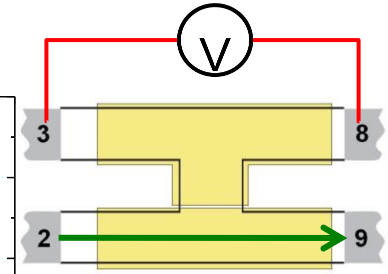
n-type

detector sweep

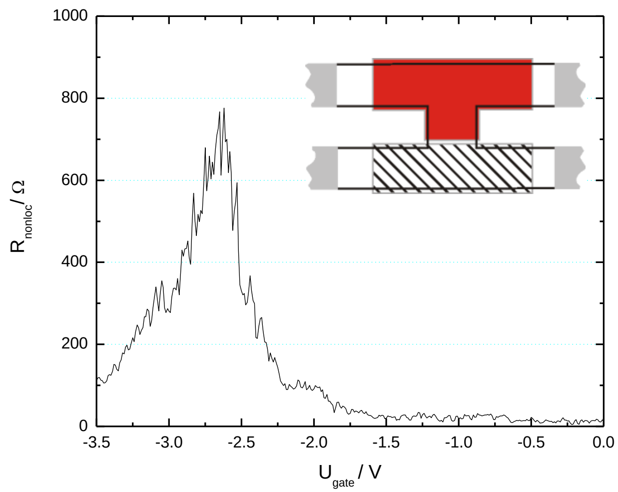


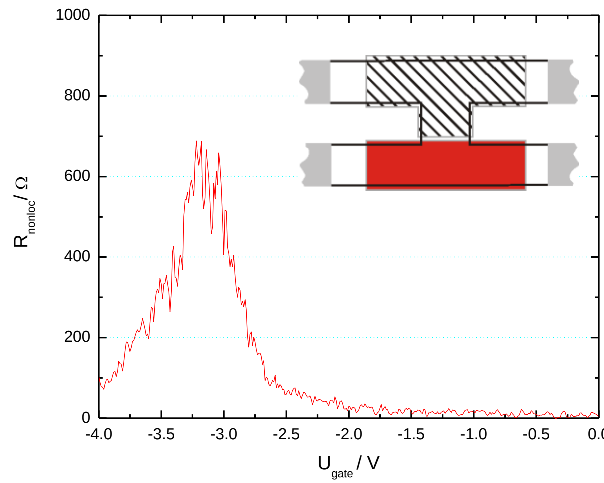




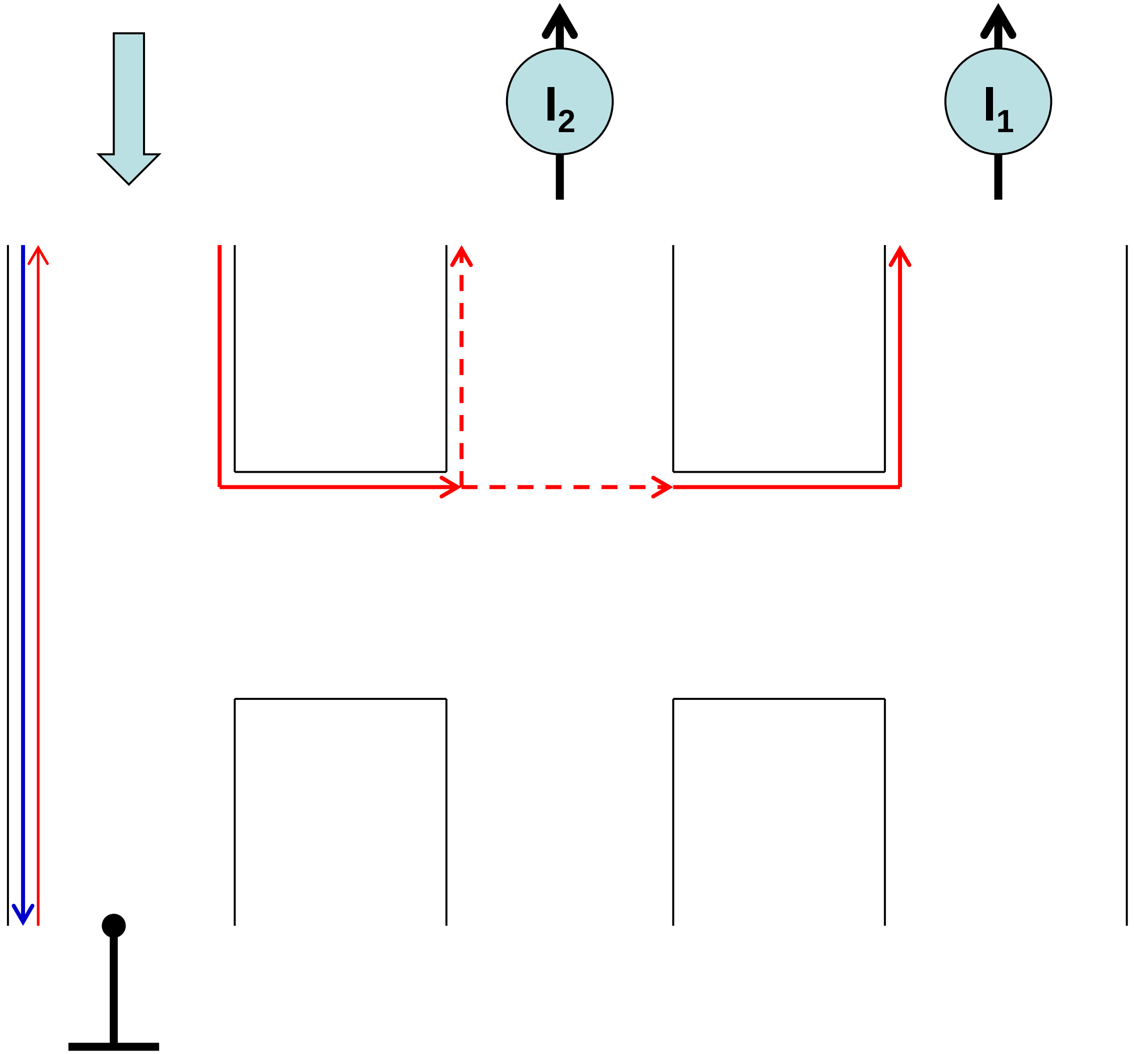


p-type

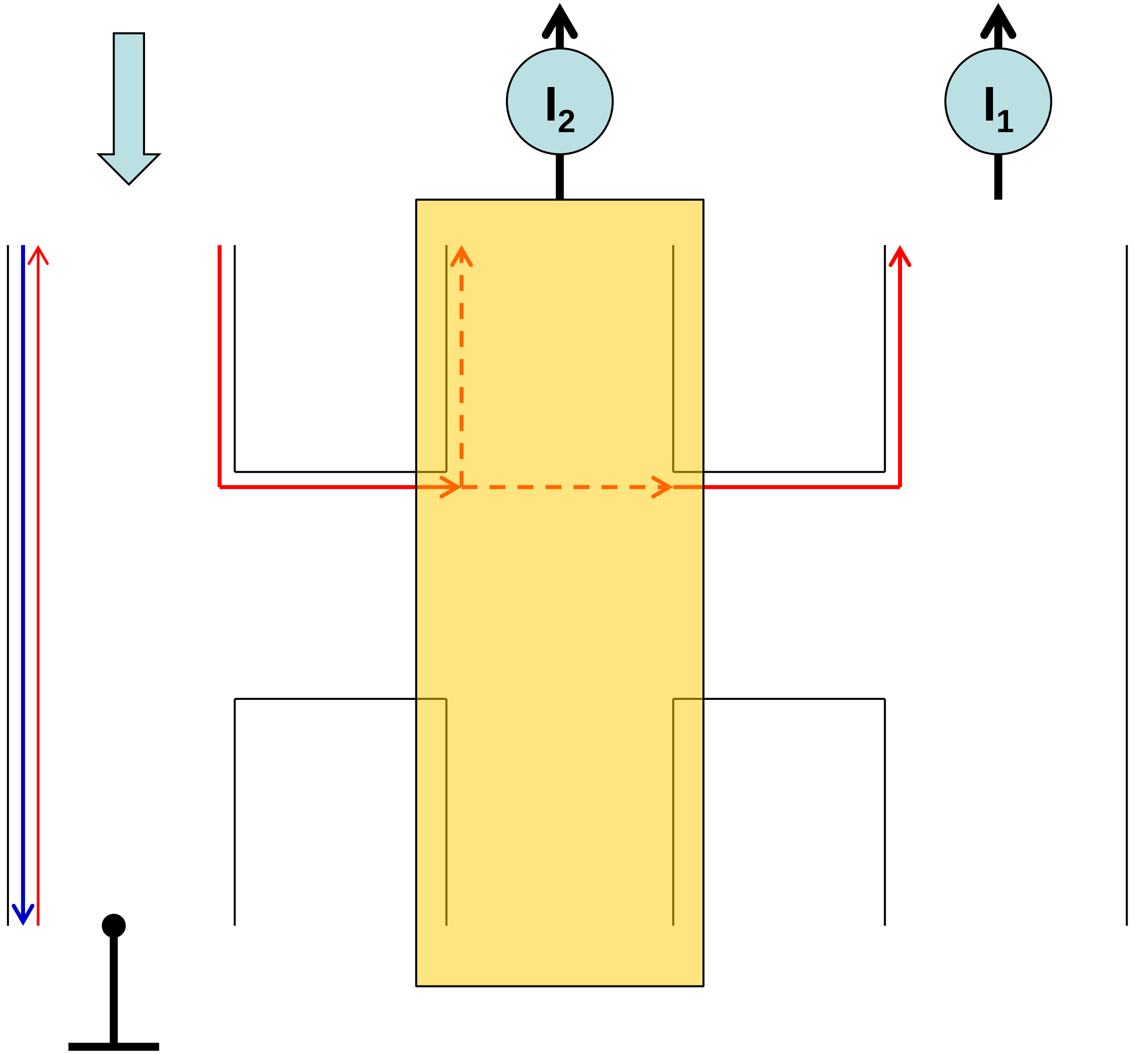




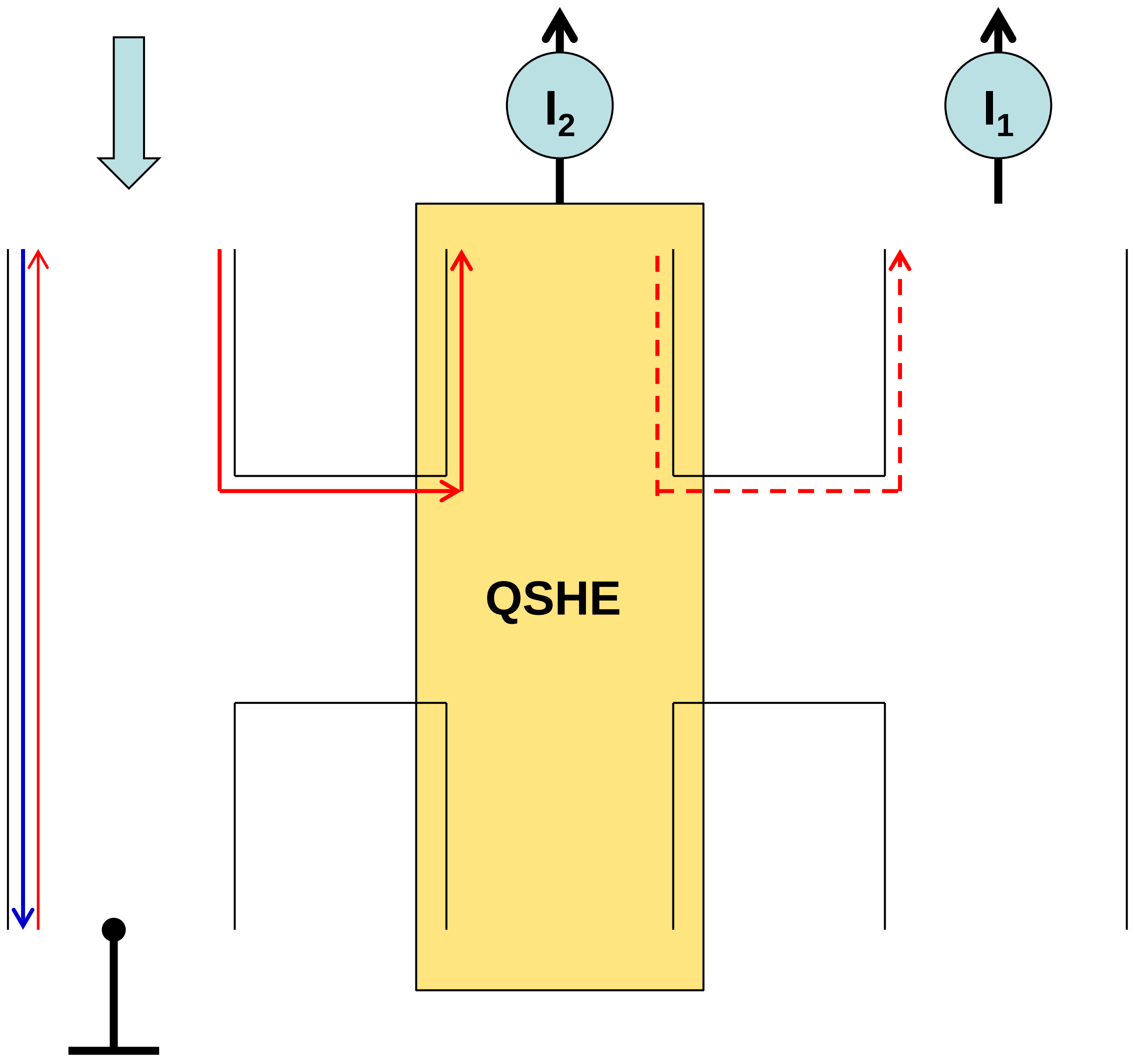
Spin-Current Switch



Spin-Current Switch

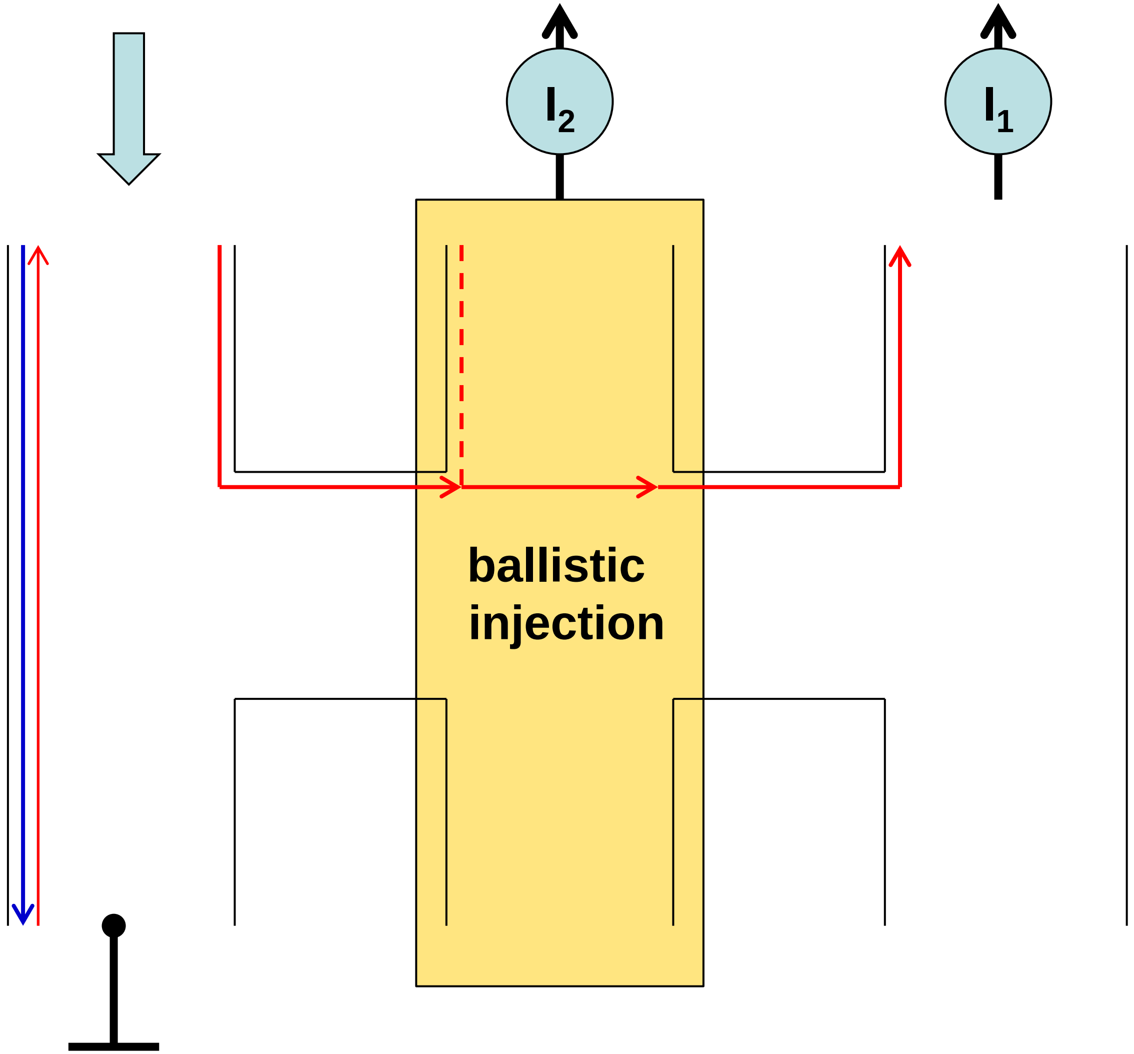


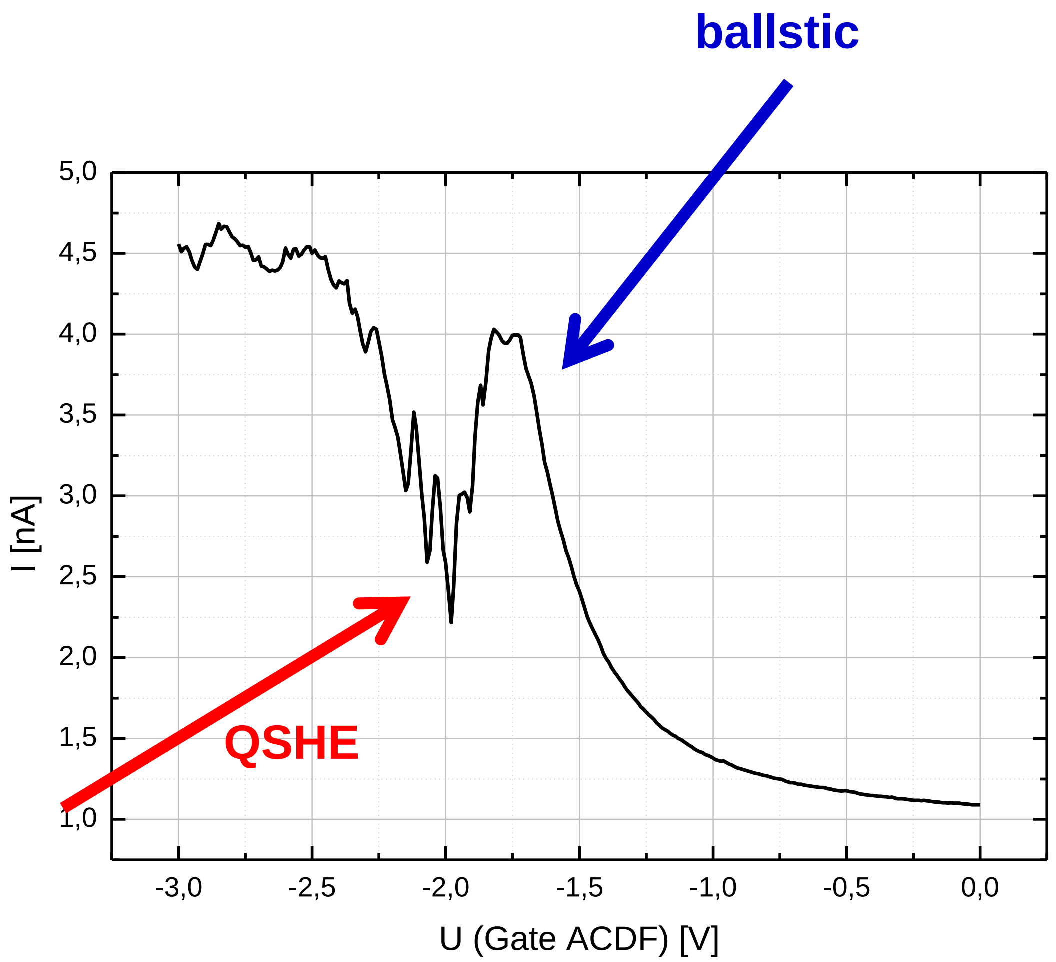
Spin-Current Switch

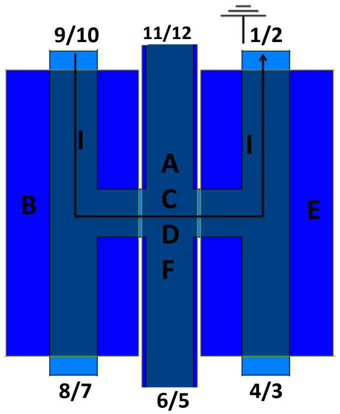


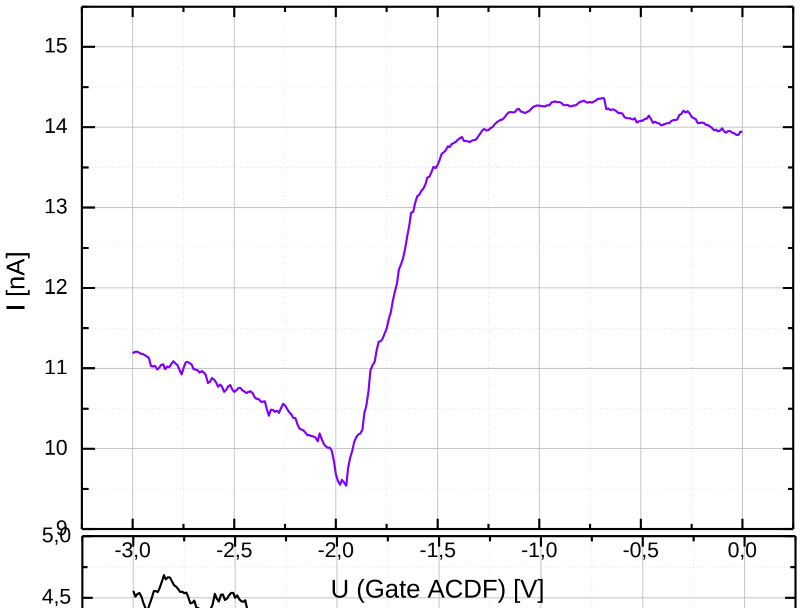
Spin-Current Switch

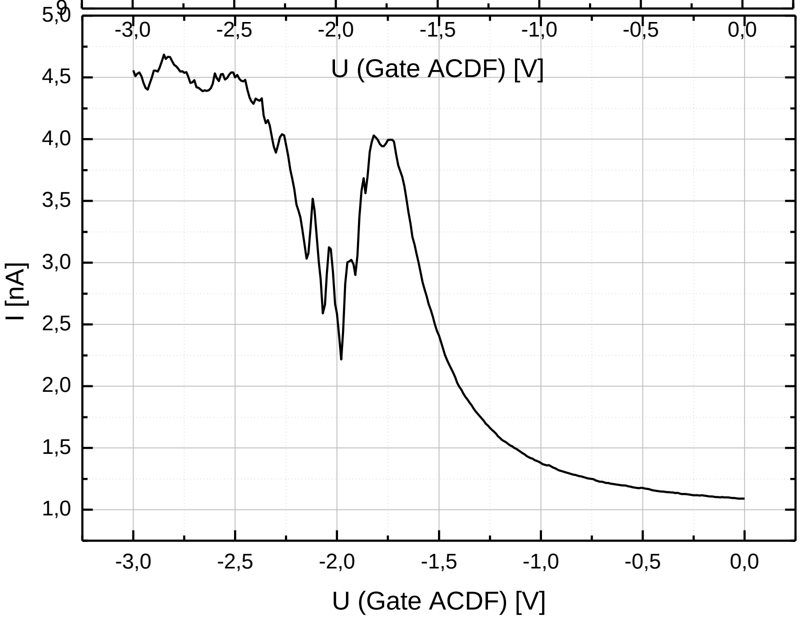
H. Buhmann

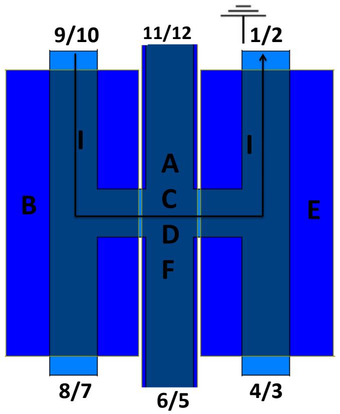



ballstic
QSHE
5,0
4,5
4,0
3,5
3,0
2,5
2,0
1,5
1,0
I [nA]
-3,0
-2,5
-2,0
-1,5
-1,0
-0,5
0,0
U (Gate ACDF) [V]

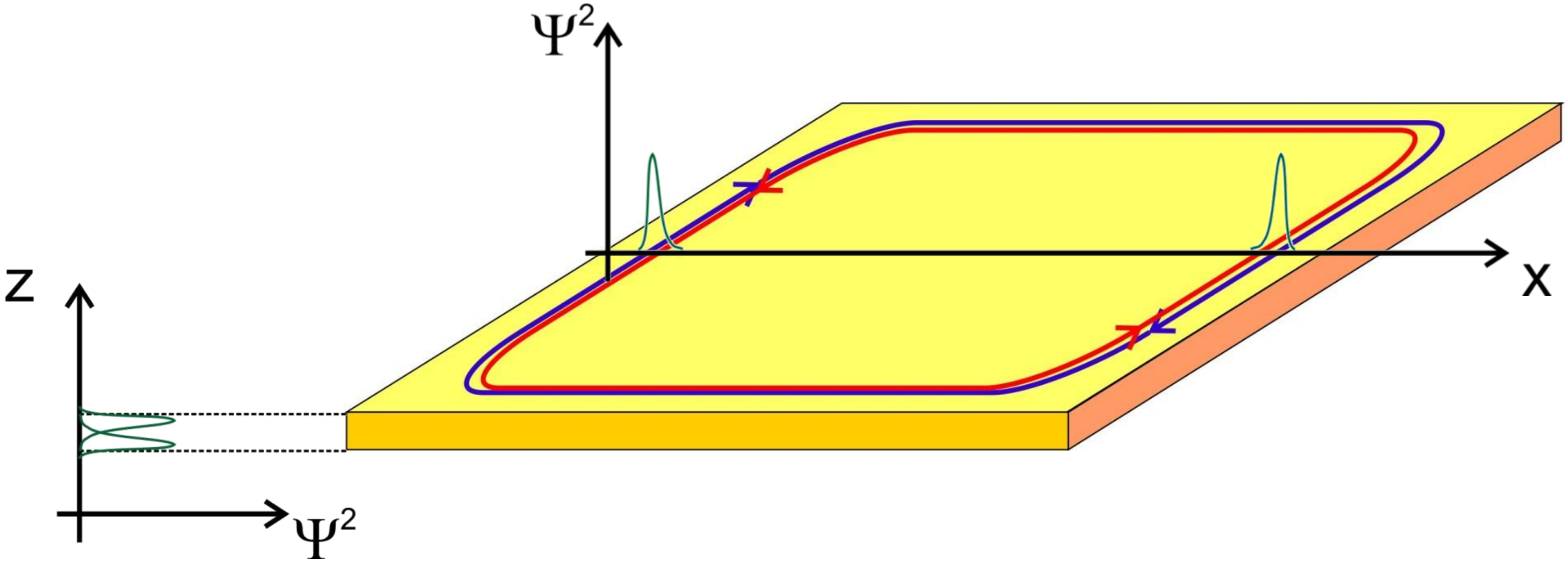

9/10
11/12
1/2
I
I
A
C
D
F
B
E
8/7
6/5
4/3

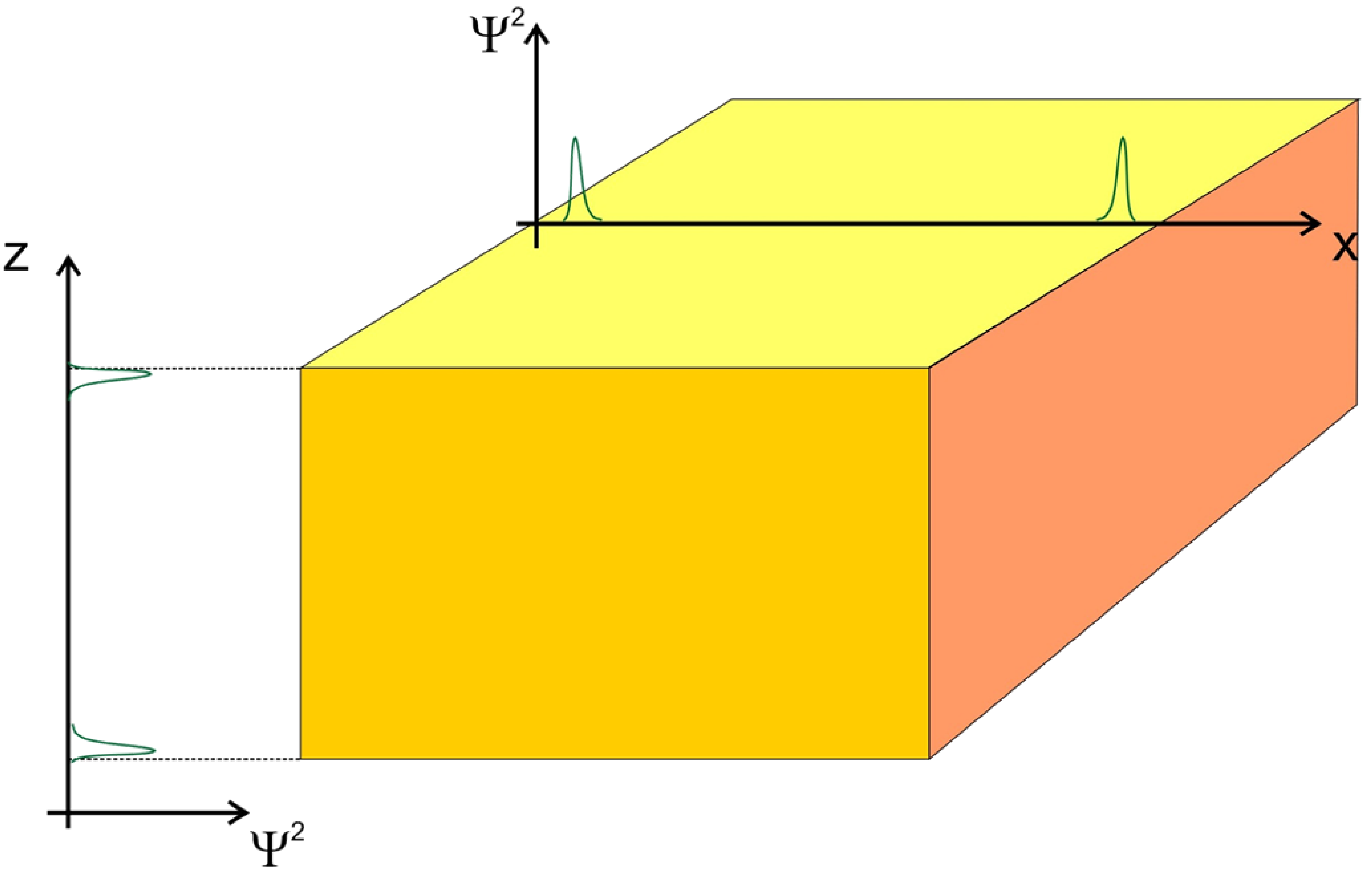

15
14
13
12
11
10
9
I [nA]
-3,0
-2,5
-2,0
-1,5
-1,0
-0,5
0,0
U (Gate ACDF) [V]


5,0
4,5
4,0
3,5
3,0
2,5
2,0
1,5
1,0
I [nA]
-3,0
-2,5
-2,0
-1,5
-1,0
-0,5
0,0
U (Gate ACDF) [V]

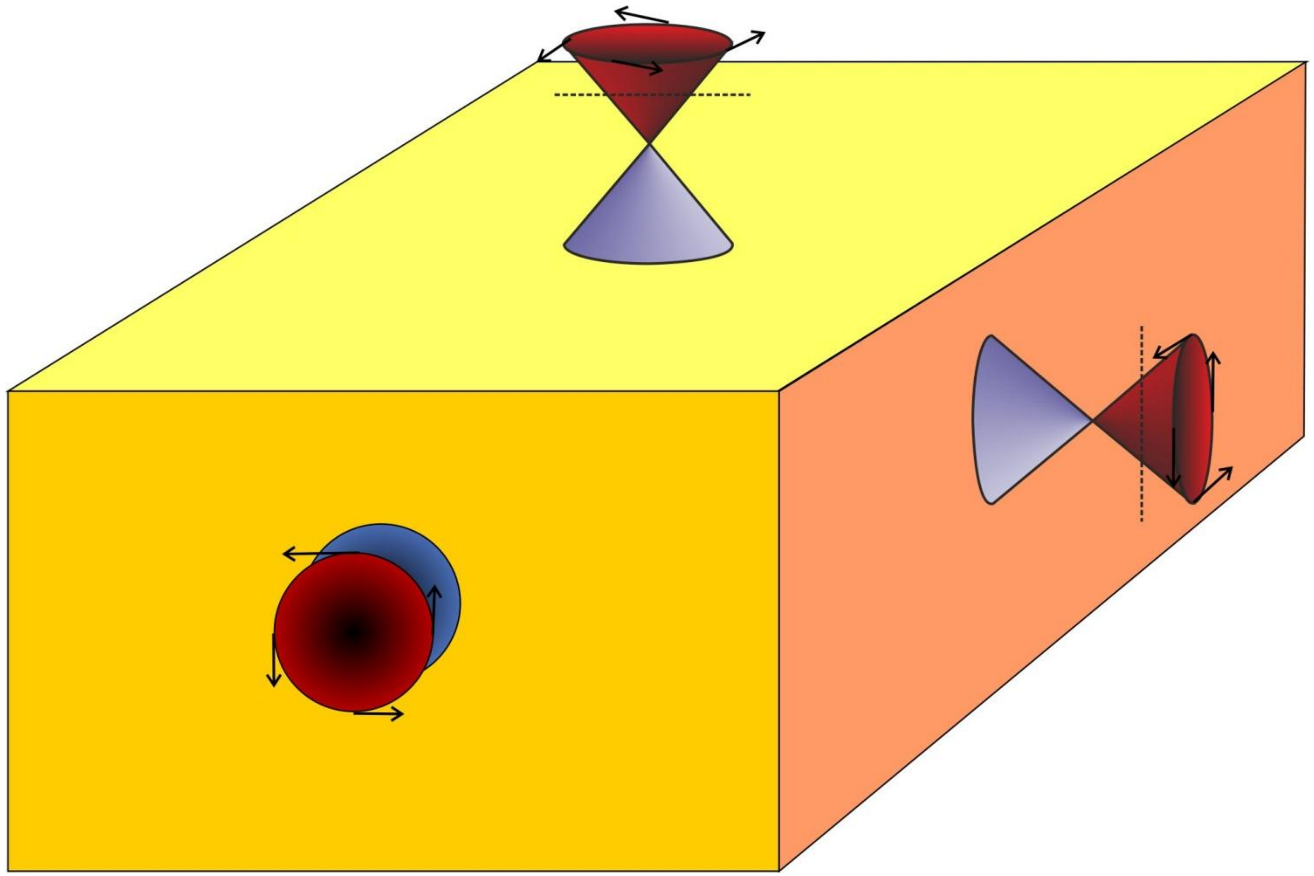

9/10
11/12
1/2
I
I
A
C
D
F
B
E
8/7
6/5
4/3

Three dimensional TI

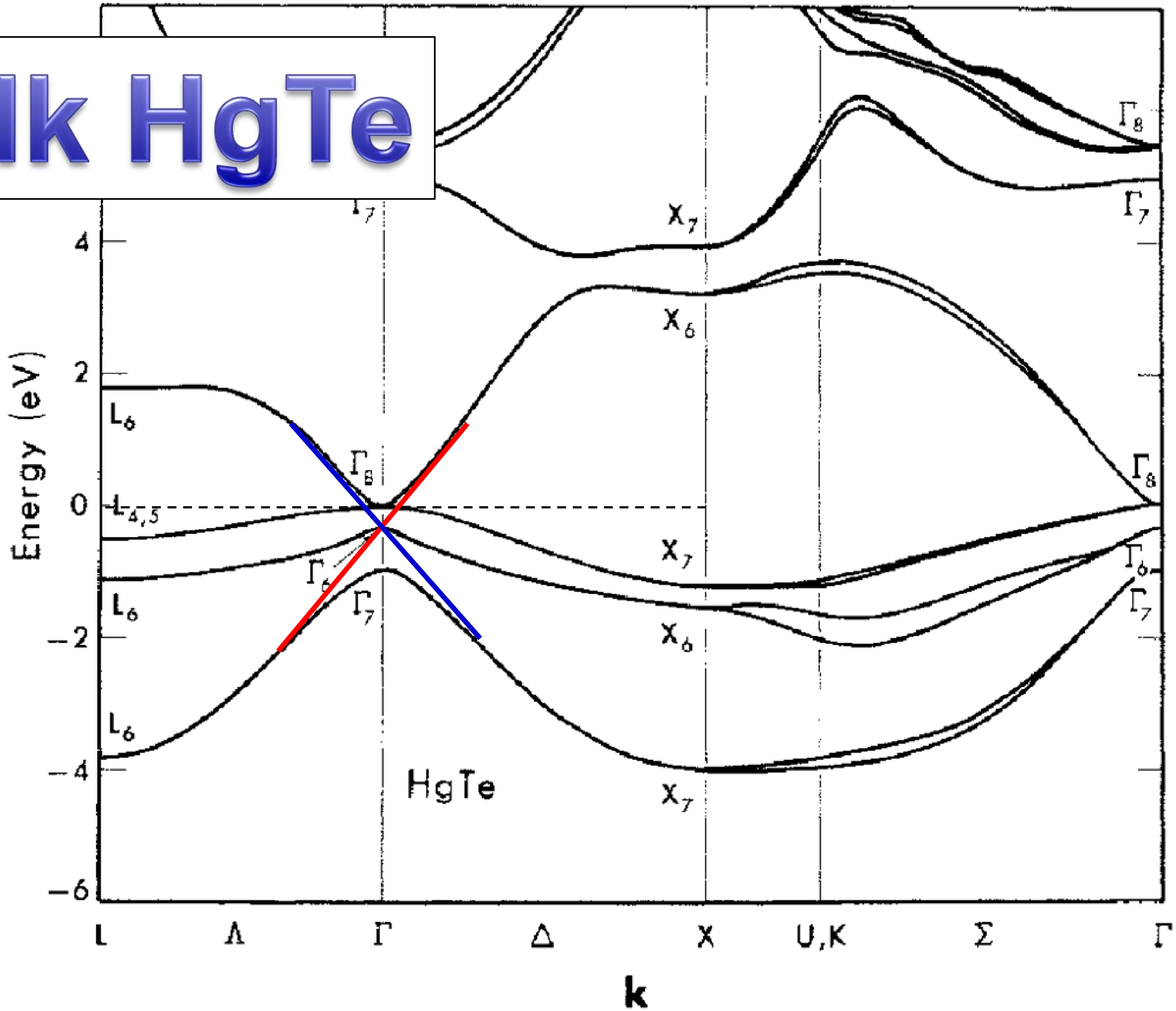

Ψ2
x
z
Ψ2


Ψ²
x
z
Ψ²

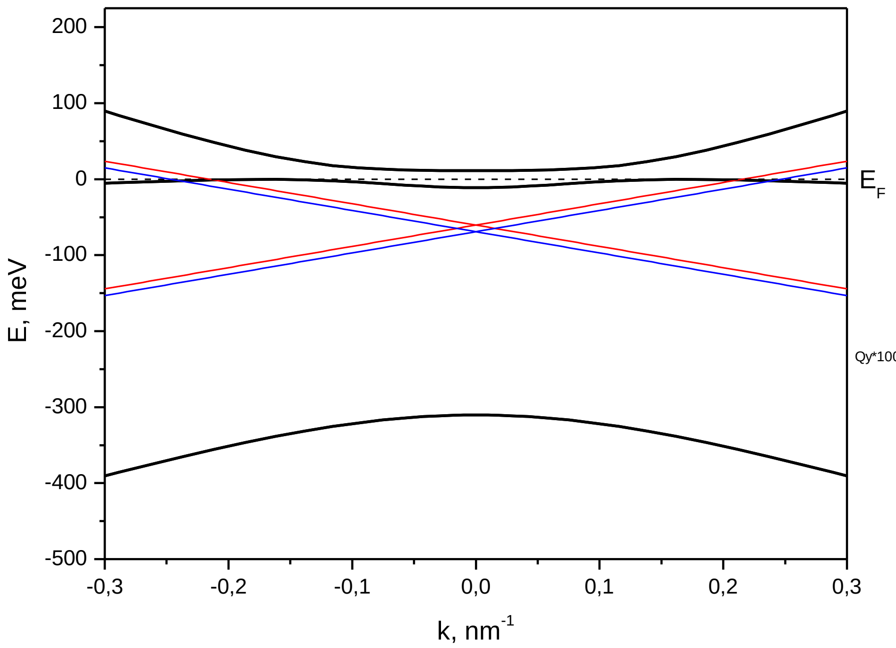
¼ Graphene



Bulk HgTe

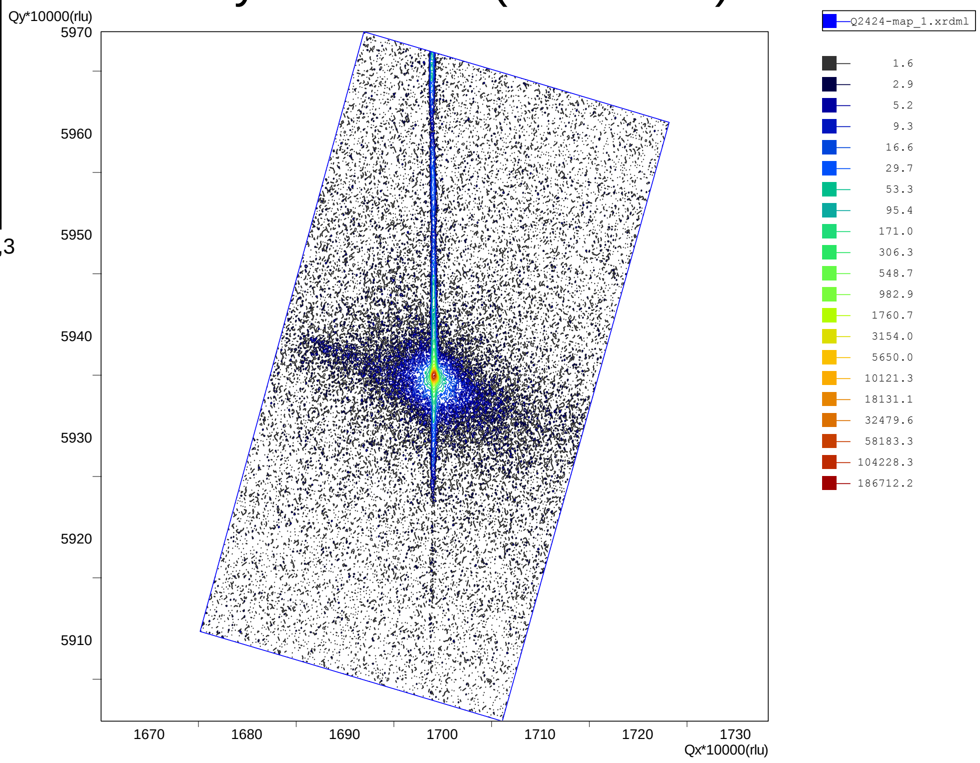

Energy (eV)
4
2
0
−2
−4
−6
L6
L4,5
L6
L6
Γ8
Γ6
Γ7
Γ7
X7
X6
X7
X6
X7
Γ8
Γ7
Γ8
Γ6
Γ7
HgTe
L
Λ
Γ
Δ
X
U,K
Σ
Γ
k

Bulk HgTe



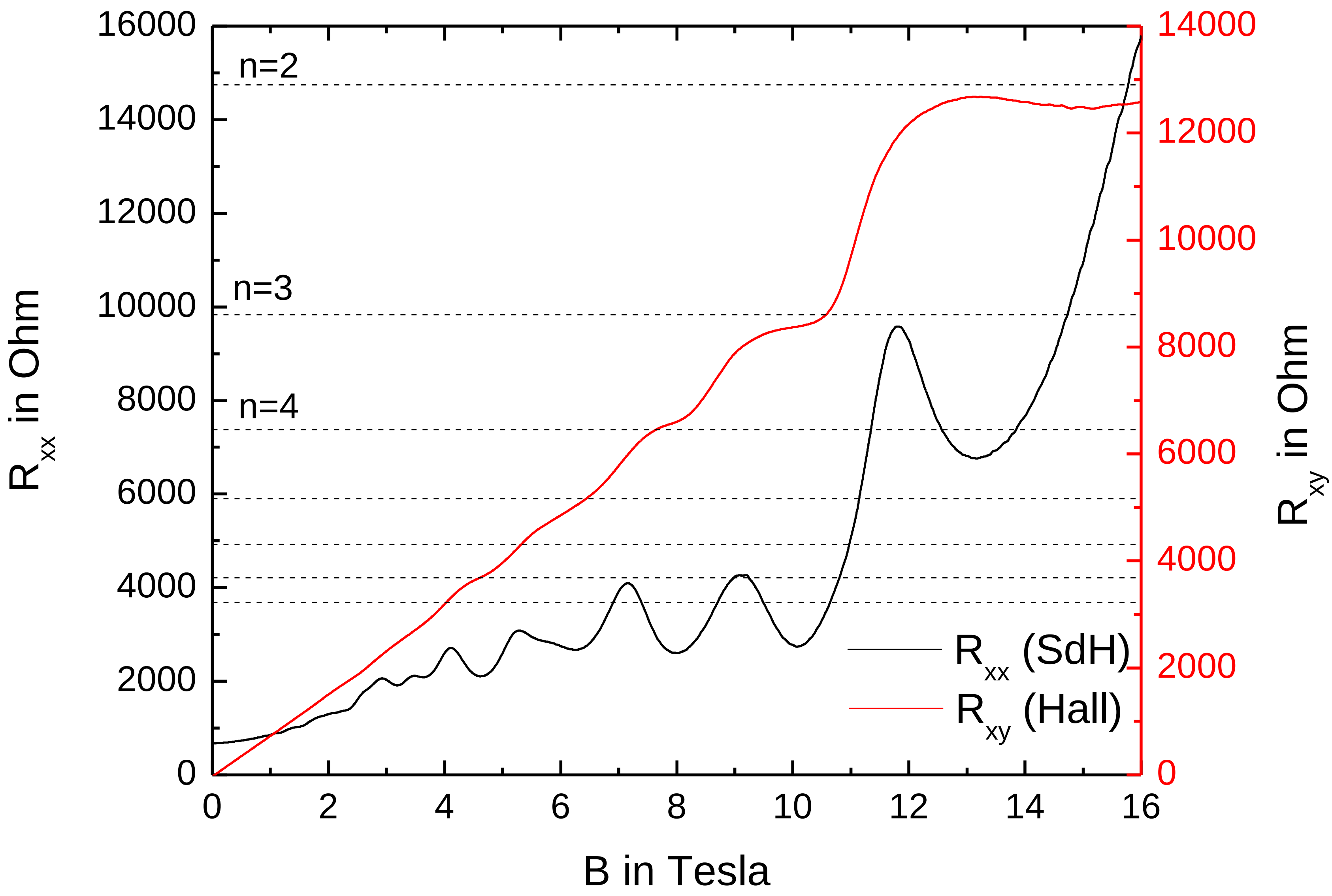
fully strained
70 nm **HgTe**
on **CdTe** substrate

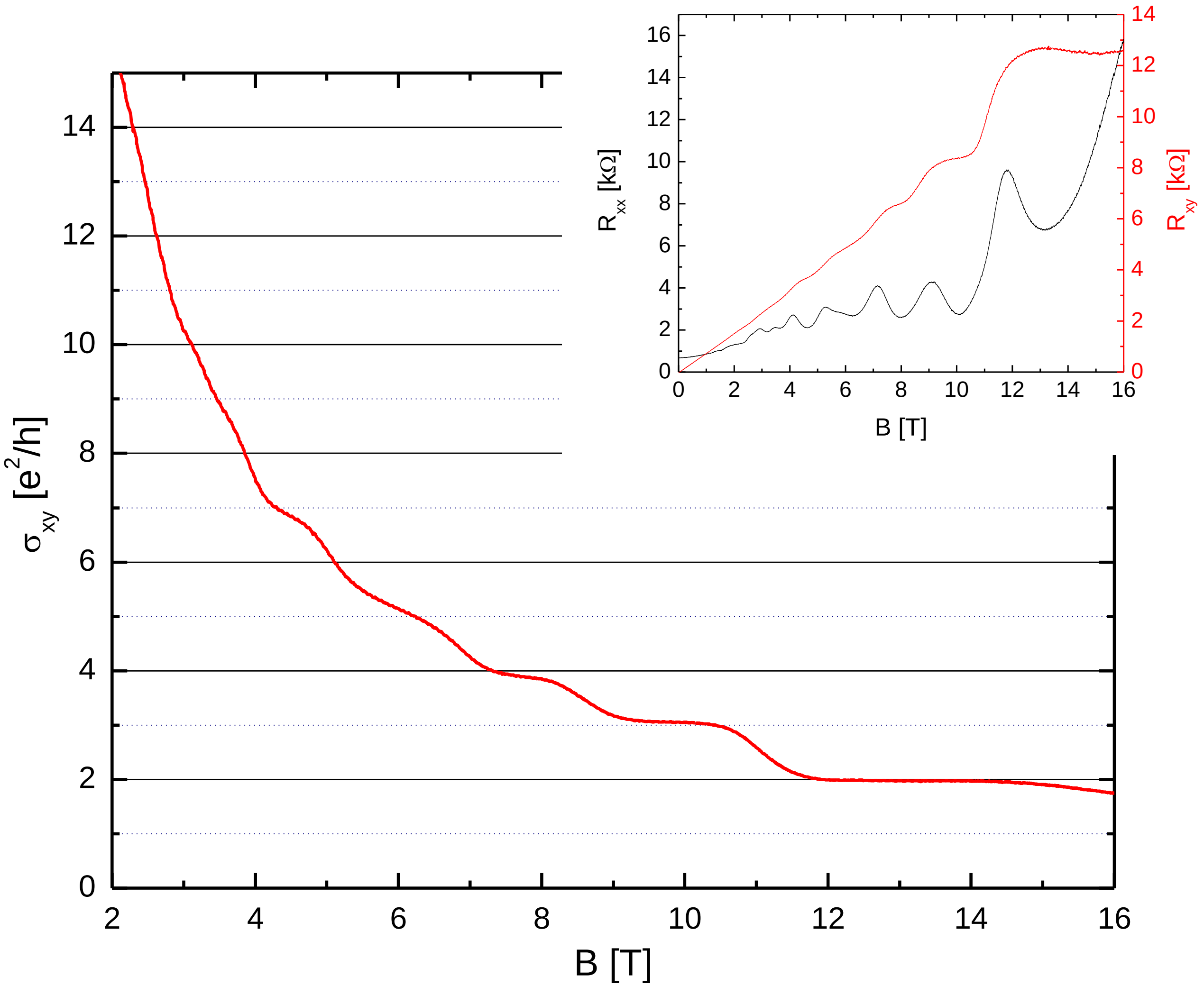
x-ray diffraction (115 reflex)



band structure

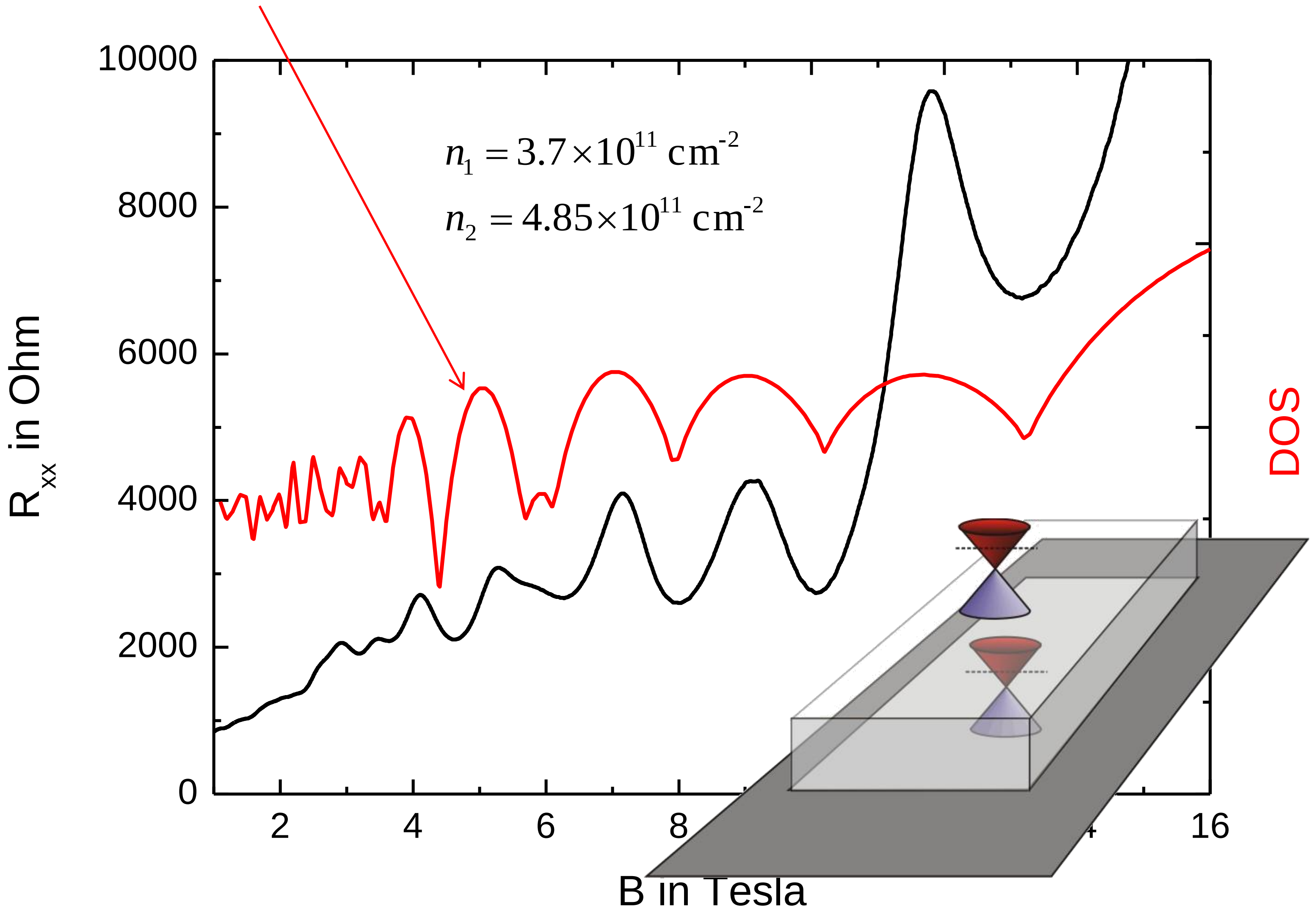
- gap openning
- two Dirac cones on different surfaces

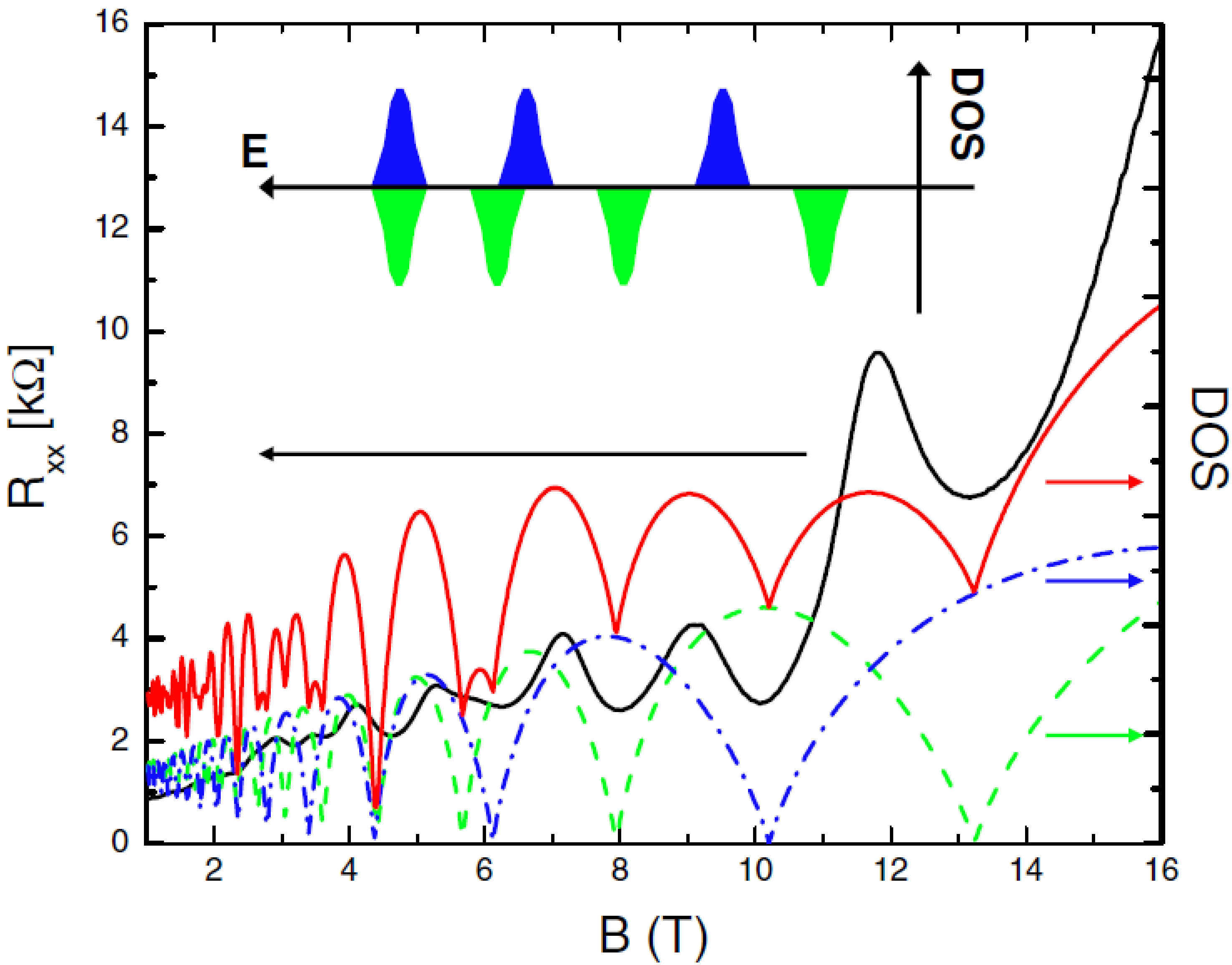

16000
14000
12000
10000
8000
6000
4000
2000
0
n=2
n=3
n=4
R_xx in Ohm
14000
12000
10000
8000
6000
4000
2000
0
R_xy in Ohm
0
2
4
6
8
10
12
14
16
B in Tesla
R_xx (SdH)
R_xy (Hall)



Brüne et al., Phys. Rev. Lett. 106, 126803 (2011)

Density of states for two independent surfaces with slightly different density





Brüne et al., Phys. Rev. Lett. 106, 126803 (2011)

Summary

- the QSH effect which consists of
  - an insulating bulk and
  - two counter propagating **spin polarized** edge channels (Kramers doublet)

- quantized edge channel transport

- the QSH effect can be used as an effective
  - spin injector and
  - spin detector

  with 100 % spin polarization properties

- Strained HgTe bulk exhibits two-dimensional surface states

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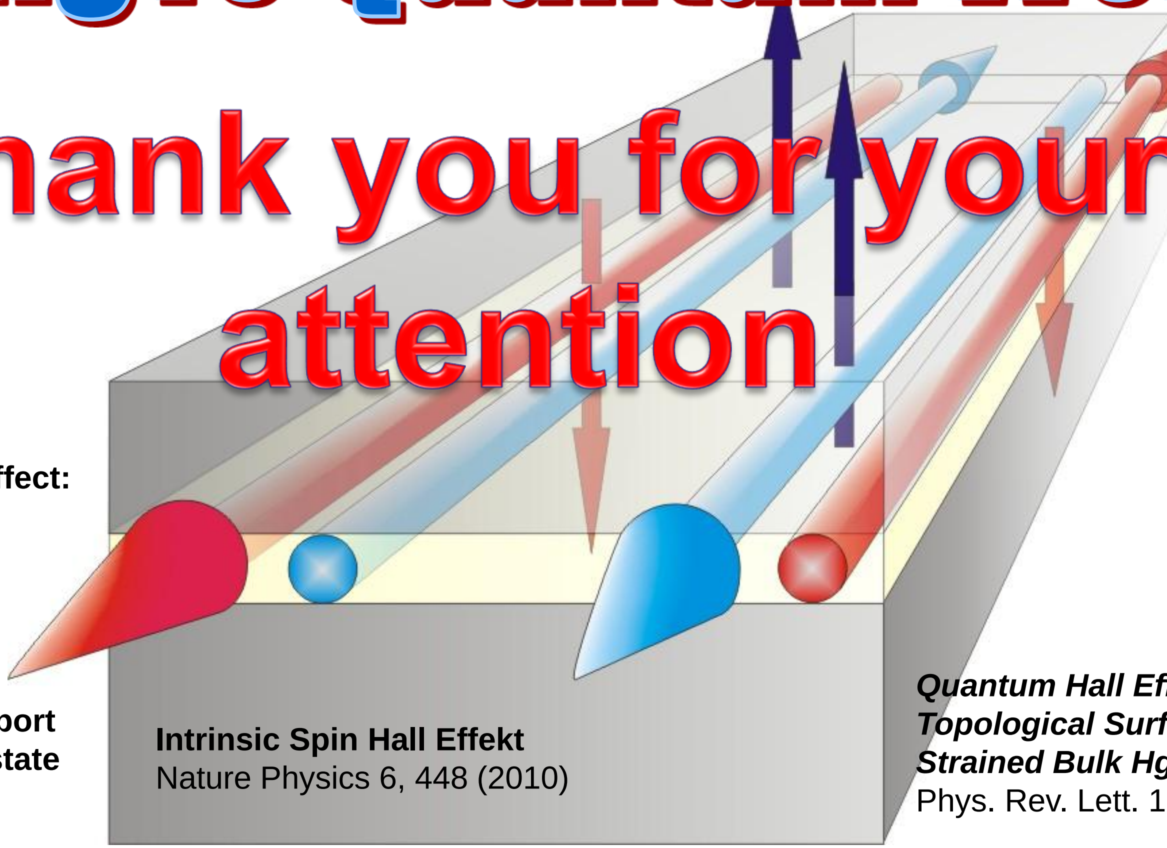
Quantum Spin Hall Effect in HgTe Quantum Wells

Thank you for your attention



Quantum Spin Hall Effekt
Science 318, 766 (2007)

The Quantum Spin Hall Effect: Theory and Experiment
J. Phys. Soc. Jap.
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Science **325**, 294 (2009)

Intrinsic Spin Hall Effekt
Nature Physics 6, 448 (2010)

Quantum Hall Effect from the Topological Surface States of Strained Bulk HgTe
Phys. Rev. Lett. 106, 126803 (2011)